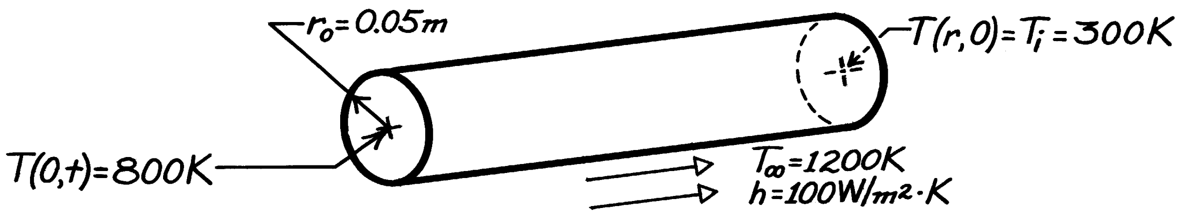
PROBLEM 5.9

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* $\left(\bar{T} = 550\ K\right)$: $r = 7832\ kg/m^3$, k = 51.2 W/m·K, c = 541 J/kg·K, $\alpha = 1.21\times10^{-5}\ m^2/s$.

ANALYSIS: The Biot number is

$$Bi = \frac{hr_o/2}{k} = \frac{100\ W/m^2 \cdot K\left(0.05\ m/2\right)}{51.2\ W/m \cdot K} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{r\,Vc}\right)t\right] = \exp\left[-\frac{4h}{r\,cD}t\right]$$

$$\ln\left(\frac{800-1200}{300-1200}\right) = -0.811 = -\frac{4\times100\ W/m^2 \cdot K}{7832\ kg/m^3\left(541\ J/kg \cdot K\right)0.1\ m}t$$

$$t = 859\ s.$$ <

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.49c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1\ \exp\left(-V_1^2 Fo\right)$$

For Bi = hr_o/k = 0.0976, Table 5.1 yields ς_1 = 0.436 and C_1 = 1.024. Hence

$$\frac{-\left(0.436\right)^2\left(1.2\times10^{-5}\ m^2/s\right)}{\left(0.05\ m\right)^2}t = \ln\left(0.434\right) = -0.835$$

$$t = 915\ s.$$

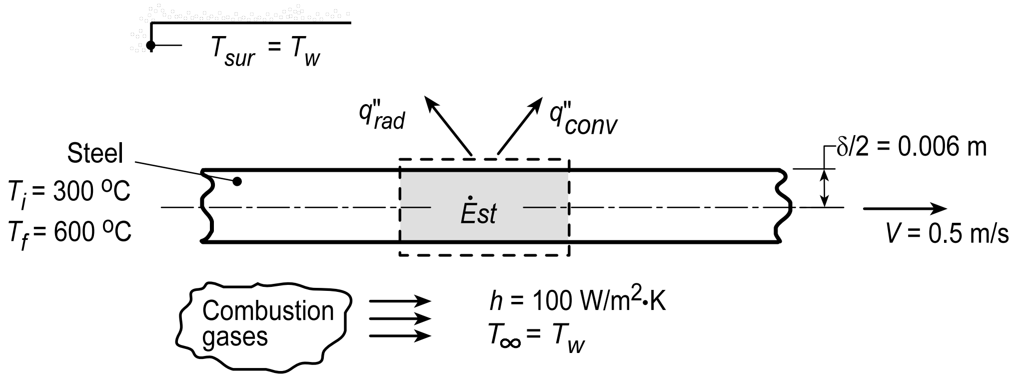
The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.

PROBLEM 5.16

KNOWN: Thickness and properties of strip steel heated in an annealing process. Furnace operating conditions.

FIND: (a) Time required to heat the strip from 300 to 600°C. Required furnace length for prescribed strip velocity (V = 0.5 m/s), (b) Effect of wall temperature on strip speed, temperature history, and radiation coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible temperature gradients in transverse direction across strip, (c) Negligible effect of strip conduction in longitudinal direction.

PROPERTIES: Steel: $\rho = 7900$ kg/m^3, $c_p = 640$ J/kg·K, k = 30 W/m·K, ε= 0.7.

ANALYSIS: (a) Considering a fixed (control) mass of the moving strip, its temperature variation with time may be obtained from an energy balance which equates the change in energy storage to heat transfer by convection and radiation. If the surface area associated with one side of the control mass is designated as A_s, $A_{s,c} = A_{s,r} = 2A_s$ and $V = \delta A_s$ in Equation 5.15, which reduces to

$$\rho c \delta \frac{dT}{dt} = -2\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

or, introducing the radiation coefficient from Equations 1.8 and 1.9 and integrating,

$$T_f - T_i = -\frac{1}{\rho c\left(\delta/2\right)}\int_o^{t_f}\left[h\left(T - T_\infty\right) + h_r\left(T - T_{sur}\right)\right]dt$$

Using the IHT *Lumped Capacitance Model* to integrate numerically with T_i = 573 K, we find that T_f = 873 K corresponds to

$t_f \approx 209$s <

in which case, the required furnace length is

$$L = Vt_f \approx 0.5\,\text{m/s} \times 209\,\text{s} \approx 105\,\text{m}$$ <

(b) For T_w = 1123 K and 1273 K, the numerical integration yields $t_f \approx 102$s and 62s respectively. Hence, for L = 105 m , $V = L/t_f$ yields

$$V\left(T_w = 1123\,\text{K}\right) = 1.03\,\text{m/s}$$

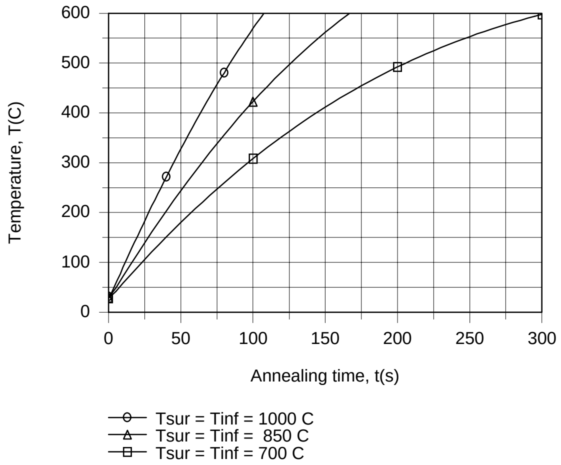
$$V\left(T_w = 1273\,\text{K}\right) = 1.69\,\text{m/s}$$ <

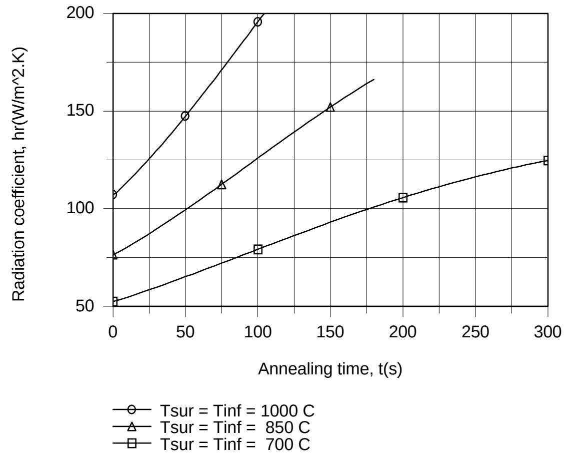
Continued...

PROBLEM 5.16 (Cont.)

which correspond to increased process rates of 106% and 238%, respectively. Clearly, productivity can be enhanced by increasing the furnace environmental temperature, albeit at the expense of increasing energy utilization and operating costs.

If the annealing process extends from 25°C (298 K) to 600°C (873 K), numerical integration yields the following results for the prescribed furnace temperatures.





As expected, the heating rate and time, respectively, increase and decrease significantly with increasing T_w. Although the radiation heat transfer rate decreases with increasing time, the coefficient h_r increases with t as the strip temperature approaches T_w.

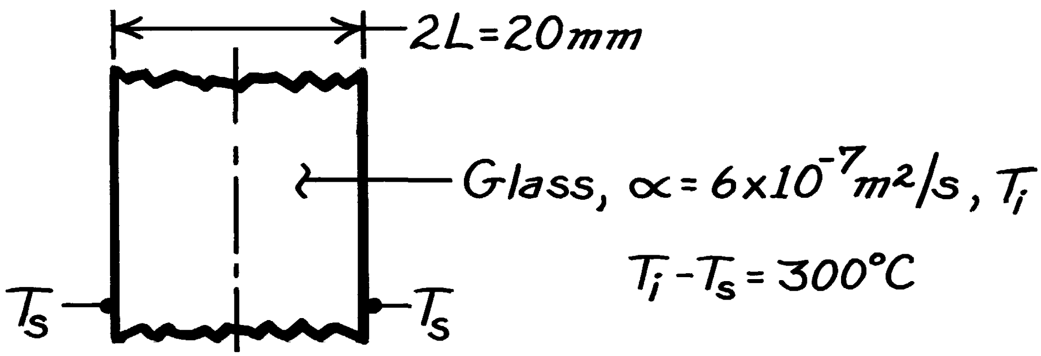
COMMENTS: To check the validity of the lumped capacitance approach, we calculate the Biot number based on a maximum cumulative coefficient of $(h + h_r) \approx 300\ W/m^2 \cdot K$. It follows that $Bi = (h + h_r)(\delta/2)/k = 0.06$ and the assumption is valid.

PROBLEM 5.40

KNOWN: Initial temperature, thickness and thermal diffusivity of glass plate. Prescribed surface temperature.

FIND: (a) Time to achieve 50% reduction in midplane temperature, (b) Maximum temperature gradient at that time.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: Prescribed surface temperature is analogous to $h \to \infty$ and $T_\infty = T_s$. Hence, Bi = ∞. Assume validity of one-term approximation to series solution for T (x,t).

(a) At the midplane,

$$\theta_o^* = \frac{T_o - T_s}{T_i - T_s} = 0.50 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

$$\zeta_1 \tan \zeta_1 = Bi = \infty \to \zeta_1 = \pi/2.$$

Hence

$$C_1 = \frac{4 \sin \zeta_1}{2\zeta_1 + \sin(2\zeta_1)} = \frac{4}{\pi} = 1.273$$

$$Fo = -\frac{\ln\left(\theta_o^*/C_1\right)}{\zeta_1^2} = 0.379$$

$$t = \frac{Fo L^2}{\alpha} = \frac{0.379\,(0.01\ \text{m})^2}{6 \times 10^{-7}\ \text{m}^2/\text{s}} = 63\ \text{s}.$$ <

(b) With $\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) \cos \zeta_1 x^*$

$$\frac{\partial T}{\partial x} = \frac{(T_i - T_s)}{L}\frac{\partial \theta^*}{\partial x^*} = -\frac{(T_i - T_s)}{L}\zeta_1 C_1 \exp\left(-\zeta_1^2 Fo\right)\sin \zeta_1 x^*$$

$$\partial T/\partial x\Big|_{max} = \partial T/\partial x\Big|_{x^*=1} = -\frac{300^\circ\text{C}}{0.01\ \text{m}}\,\frac{\pi}{2}\,0.5 = -2.36 \times 10^4\ ^\circ\text{C/m}.$$ <

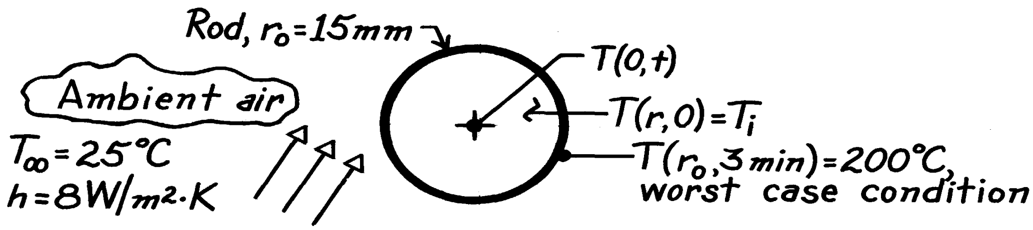
COMMENTS: Validity of one-term approximation is confirmed by Fo > 0.2.

PROBLEM 5.53

KNOWN: Long plastic rod of diameter D heated uniformly in an oven to T_i and then allowed to convectively cool in ambient air (T_∞, h) for a 3 minute period. Minimum temperature of rod should not be less than 200°C and the maximum-minimum temperature within the rod should not exceed 10°C.

FIND: Initial uniform temperature T_i to which rod should be heated. Whether the 10°C internal temperature difference is exceeded.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Uniform and constant convection coefficients.

PROPERTIES: Plastic rod (given): k = 0.3 W/m·K, ρc_p = 1040 kJ/m^3·K.

ANALYSIS: For the worst case condition, the rod cools for 3 minutes and its outer surface is at least 200°C in order that the subsequent pressing operation will be satisfactory. Hence,

$$\mathrm{Bi} = \frac{h r_o}{k} = \frac{8\ \mathrm{W/m^2 \cdot K} \times 0.015\ \mathrm{m}}{0.3\ \mathrm{W/m \cdot K}} = 0.40$$

$$\mathrm{Fo} = \frac{a\, t}{r_o^2} = \frac{k}{r\, c_p} \cdot \frac{t}{r_o^2} = \frac{0.3\ \mathrm{W/m \cdot K}}{1040 \times 10^3\ \mathrm{J/m^3 \cdot K}} \times \frac{3 \times 60\mathrm{s}}{(0.015\ \mathrm{m})^2} = 0.2308.$$

Using Eq. 5.49a and $z_1 = 0.8516$ rad and C_1 = 1.0932 from Table 5.1,

$$q^* = \frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = C_1 J_0\left(z_1 r_o^*\right) \exp\left(-z_1^2 \mathrm{Fo}\right).$$

With $r_o^* = 1$, from Table B.4, $J_0(z_1 \times 1) = J_0(0.8516) = 0.8263$, giving

$$\frac{200 - 25}{T_i - 25} = 1.0932 \times 0.8263 \exp\left(-0.8516^2 \times 0.2308\right) \qquad T_i = 254°\mathrm{C}.$$ <

At this time (3 minutes) what is the difference between the center and surface temperatures of the rod? From Eq. 5.49b,

$$\frac{q^*}{q_o} = \frac{T(r_o, t) - T_\infty}{T(0,t) - T_\infty} = \frac{200 - 25}{T(0,t) - 25} = J_0\left(z_1 r_o^*\right) = 0.8263$$

which gives T(0,t) = 237°C. Hence,

$$\Delta T = T(0,180\mathrm{s}) - T(r_o, 180\mathrm{s}) = (237 - 200)^\circ \mathrm{C} = 37°\mathrm{C}.$$ <

Hence, the desired max-min temperature difference sought (10°C) is not achieved.

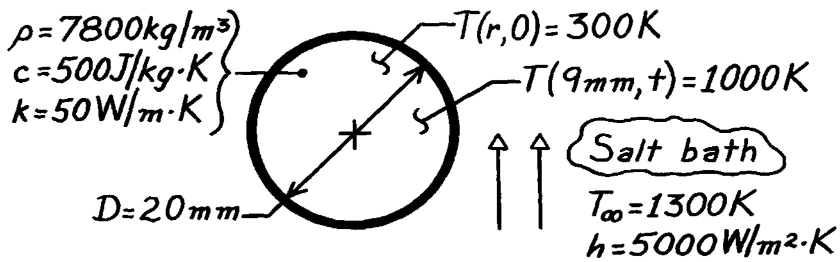
COMMENTS: ΔT could be reduced by decreasing the cooling rate; however, h can not be made much smaller. Two solutions are (a) increase ambient air temperature and (b) non-uniformly heat rod in oven by controlling its residence time.

PROBLEM 5.58

KNOWN: A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with T > 1000K.

FIND: Time required to harden outer layer of 1mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo ≥ 0.2.

ANALYSIS: Since any location within the ball whose temperature exceeds 1000K will be hardened, the problem is to find the time when the location r = 9mm reaches 1000K. Then a 1mm outer layer will be hardened. Begin by finding the Biot number.

$$\mathrm{Bi} = \frac{h\,r_o}{k} = \frac{5000\ \mathrm{W/m^2\cdot K}\,(0.020\mathrm{m}/2)}{50\ \mathrm{W/m\cdot K}} = 1.00.$$

Using the one-term approximate solution for a sphere, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2}\ln\left[\theta^*/C_1\frac{1}{\zeta_1 r^*}\sin\left(\zeta_1 r^*\right)\right].$$

From Table 5.1 with Bi = 1.00, for the sphere find $\zeta_1 = 1.5708$ rad and C_1 = 1.2732. With r* = r/r_o = (9mm/10mm) = 0.9, substitute numerical values.

$$\mathrm{Fo} = \frac{-1}{(1.5708)^2}\ln\left[\frac{(1000-1300)\mathrm{K}}{(300-1300)\mathrm{K}}/1.2732\frac{1}{1.5708\times 0.9}\sin(1.5708\times 0.9\ \mathrm{rad})\right] = 0.441.$$

From the definition of the Fourier number with α = k/ρc,

$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo}\cdot r_o^2\frac{\rho\, c}{k} = 0.441\times\left[\frac{0.020\mathrm{m}}{2}\right]^2 7800\frac{\mathrm{kg}}{\mathrm{m^3}}\times 500\frac{\mathrm{J}}{\mathrm{kg\cdot K}}/50\ \mathrm{W/m\cdot K} = 3.4\mathrm{s}. \qquad <$$

COMMENTS: (1) Note the very short time required to harden the ball. At this time it can be easily shown the center temperature is T(0,3.4s) = 871 K.

(2) The Heisler charts can also be used. From Fig. D.8, with Bi^{-1} = 1.0 and r/r_o = 0.9, read θ/θ_o = 0.69(±0.03). Since

$$\theta = T - T_\infty = 1000 - 1300 = -300\mathrm{K} \qquad \theta_i = T_i - T_\infty = -1000\mathrm{K}$$

it follows that

$$\frac{\theta}{\theta_i} = 0.30. \quad \text{Since} \quad \frac{\theta}{\theta_i} = \frac{\theta}{\theta_o}\cdot\frac{\theta_o}{\theta_i}, \quad \text{then} \quad \frac{\theta}{\theta_i} = 0.69\ \frac{\theta_o}{\theta_i}$$

and $\theta_o/\theta_i = 0.30/0.69 = 0.43\ (\pm 0.02)$.

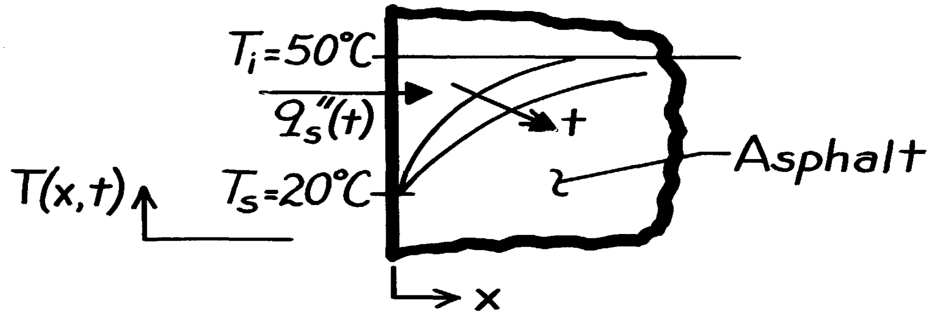
From Fig. D.7 at θ_o/θ_i=0.43, Bi^{-1}=1.0, read Fo = 0.45 (±0.03) and t = 3.5 (±0.2)s. Note the use of tolerances associated with reading the charts to ±5%.

PROBLEM 5.71

KNOWN: Asphalt pavement, initially at 50°C, is suddenly exposed to a rainstorm reducing the surface temperature to 20°C.

FIND: Total amount of energy removed (J/m^2) from the pavement for a 30 minute period.

SCHEMATIC:



ASSUMPTIONS: (1) Asphalt pavement can be treated as a semi-infinite solid, (2) Effect of rainstorm is to suddenly reduce the surface temperature to 20°C and is maintained at that level for the period of interest.

PROPERTIES: *Table A-3*, Asphalt (300K): $\rho = 2115\ kg/m^3$, c = 920 J/kg·K, k = 0.062 W/m·K.

ANALYSIS: This solution corresponds to Case 1, Figure 5.7, and the surface heat flux is given by Eq. 5.58 as

$$q_s''(t) = k\left(T_s - T_i\right)/\left(\pi\alpha\ t\right)^{1/2} \qquad (1)$$

The energy into the pavement over a period of time is the integral of the surface heat flux expressed as

$$Q'' = \int_0^t q_s''(t)\,dt. \qquad (2)$$

Note that $q_s''(t)$ is into the solid and, hence, Q represents energy into the solid. Substituting Eq. (1) for $q_s''(t)$ into Eq. (2) and integrating find

$$Q'' = k\left(T_s - T_i\right)/\left(\pi\alpha\right)^{1/2}\int_0^t t^{-1/2}dt = \frac{k\left(T_s - T_i\right)}{\left(\pi\alpha\right)^{1/2}}\times 2\ t^{1/2}. \qquad (3)$$

Substituting numerical values into Eq. (3) with

$$\alpha = \frac{k}{\rho c} = \frac{0.062\ W/m\cdot K}{2115\ kg/m^3 \times 920\ J/kg\cdot K} = 3.18\times10^{-8} m^2/s$$

find that for the 30 minute period,

$$Q'' = \frac{0.062\ W/m\cdot K\ \left(20-50\right)K}{\left(\pi\times 3.18\times10^{-8} m^2/s\right)^{1/2}}\times 2\left(30\times 60 s\right)^{1/2} = -4.99\times10^5\ J/m^2. \qquad <$$

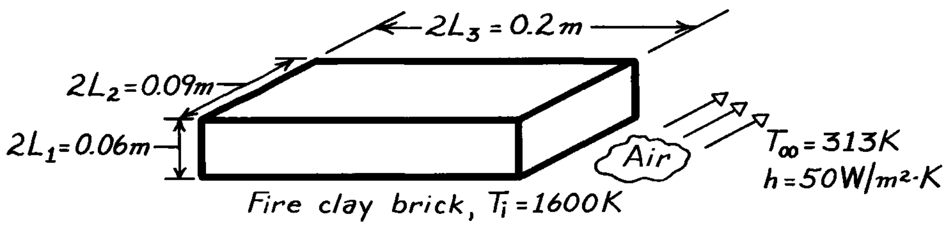
COMMENTS: Note that the sign for Q'' is negative implying that energy is removed from the solid.

PROBLEM 5.88

KNOWN: Initial temperature of fire clay brick which is cooled by convection.

FIND: Center and corner temperatures after 50 minutes of cooling.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium with constant properties, (2) Negligible radiation effects.

PROPERTIES: *Table A-3*, Fire clay brick (900K): ρ = 2050 kg/m^3, k = 1.0 W/m·K, c_p = 960 J/kg·K. $\alpha = 0.51 \times 10^{-6}$ m^2/s.

ANALYSIS: From Fig. 5.11(h), the center temperature is given by

$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} = P_1(0,t) \times P_2(0,t) \times P_3(0,t)$$

where P_1, P_2 and P_3 must be obtained from Fig. D.1.

$$L_1 = 0.03\text{m}: \qquad Bi_1 = \frac{h\,L_1}{k} = 1.50 \qquad Fo_1 = \frac{\alpha\,t}{L_1^2} = 1.70$$

$$L_2 = 0.045\text{m}: \qquad Bi_2 = \frac{h\,L_2}{k} = 2.25 \qquad Fo_2 = \frac{\alpha\,t}{L_2^2} = 0.756$$

$$L_3 = 0.10\text{m}: \qquad Bi_3 = \frac{h\,L_3}{k} = 5.0 \qquad Fo_3 = \frac{\alpha\,t}{L_3^2} = 0.153$$

Hence from Fig. D.1,

$$P_1(0,t) \approx 0.22 \qquad P_2(0,t) \approx 0.50 \qquad P_3(0,t) \approx 0.85.$$

Hence,
$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} \approx 0.22 \times 0.50 \times 0.85 = 0.094$$

and the center temperature is

$$T(0,0,0,t) \approx 0.094(1600 - 313)\text{K} + 313\text{K} = 434\text{K}.$$ <

Continued …..

PROBLEM 5.88 (Cont.)

The corner temperature is given by

$$\frac{T(L_1, L_2, L_3, t) - T_\infty}{T_i - T_\infty} = P(L_1, t) \times P(L_2, t) \times P(L_3, t)$$

where

$$P(L_1, t) = \frac{\theta(L_1, t)}{\theta_o} \cdot P_1(0, t), \text{ etc.}$$

and similar forms can be written for L_2 and L_3. From Fig. D.2,

$$\frac{\theta(L_1, t)}{\theta_o} \approx 0.55 \qquad \frac{\theta(L_2, t)}{\theta_o} \approx 0.43 \qquad \frac{\theta(L_3, t)}{\theta_o} \approx 0.25.$$

Hence,

$$P(L_1, t) \approx 0.55 \times 0.22 = 0.12$$
$$P(L_2, t) \approx 0.43 \times 0.50 = 0.22$$
$$P(L_3, t) \approx 0.85 \times 0.25 = 0.21$$

and

$$\frac{T(L_1, L_2, L_3, t) - T_\infty}{T_i - T_\infty} \approx 0.12 \times 0.22 \times 0.21 = 0.0056$$

or

$$T(L_1, L_2, L_3, t) \approx 0.0056(1600 - 313)\text{K} + 313\text{K}.$$

The corner temperature is then

$$T(L_1, L_2, L_3, t) \approx 320\text{K}.$$ <

COMMENTS: (1) The foregoing temperatures are overpredicted by ignoring radiation, which is significant during the early portion of the transient.

(2) Note that, if the time required to reach a certain temperature were to be determined, an iterative approach would have to be used. The foregoing procedure would be used to compute the temperature for an assumed value of the time, and the calculation would be repeated until the specified temperature were obtained.