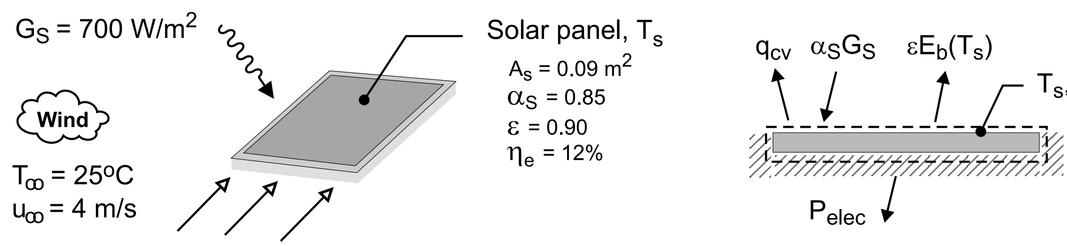
PROBLEM 7.18

KNOWN: Square solar panel with an area of 0.09 m^2 has solar-to-electrical power conversion efficiency of 12%, solar absorptivity of 0.85, and emissivity of 0.90. Panel experiences a 4 m/s breeze with an air temperature of 25°C and solar insolation of 700 W/m^2.

FIND: Estimate the temperature of the solar panel for: (a) The operating condition *(on)* described above when the panel is producing power, and (b) The *off* condition when the solar array is inoperative. Will the panel temperature increase, remain the same or decrease, all other conditions remaining the same?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The backside of the panel experiences no heat transfer, (3) Sky irradiation is negligible, and (4) Wind is in parallel, fully turbulent flow over the panel.

PROPERTIES: *Table A-4*, Air (Assume T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) Perform an energy balance on the panel as represented in the schematic above considering convection, absorbed insolation, emission and generated electrical power.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{cv} + \left[\alpha_S G_S - \varepsilon\sigma T_s^4\right] A_s - P_{elec} = 0 \qquad (1)$$

Using the convection rate equation and power conversion efficiency,

$$q_{cv} = \bar{h}_L A_s \left(T_s - T_\infty\right) \qquad P_{elec} = \eta_e \alpha_S G_S A_s \qquad (2,3)$$

The average convection coefficient for fully turbulent conditions is

$$\overline{Nu}_L = \bar{h}L/k = 0.037\ Re_L^{4/5}\ Pr^{1/3}$$

$$Re_L = u_\infty L/\nu = 4\,m/s \times 0.3\,m/15.89\times10^{-6} m^2/s = 7.49\times10^4$$

$$\bar{h}_L = \left(0.0263\ W/m\cdot K/0.3m\right)\times0.037\times\left(7.49\times10^4\right)^{4/5}\left(0.707\right)^{1/3}$$

$$\bar{h}_L = 23.0\ W/m^2\cdot K$$

Substituting numerical values in Eq. (1) using Eqs. (2 and 3) and dividing through by A_s, find T_s.

Continued …..

$$23\ \mathrm{W/m^2 \cdot K}\left(T_s - 298\right)\mathrm{K} + 0.85 \times 700\ \mathrm{W/m^2} - 0.90 \times 5.67 \times 10^{-8}\mathrm{W/m^2 \cdot K^4}\,T_s^4$$
$$-0.12\left[0.85 \times 700\ \mathrm{W/m^2}\right] = 0 \qquad (4)$$

$$T_s = 302.2\ \mathrm{K} = 29.2°\mathrm{C}$$ <

(b) If the solar array becomes inoperable *(off)* for reason of wire bond failures or the electrical circuit to the battery is opened, the P_{elec} term in the energy balance of Eq. (1) is zero. Using Eq. (4) with η_e = 0, find

$$T_s = 31.7°\mathrm{C}$$ <

COMMENTS: (1) Note how the electrical power P_{elec} is represented by the $\dot{E}_{gen}$ term in the energy balance. Recall from Section 1.2 that $\dot{E}_{gen}$ is associated with conversion *from* some form of energy *to* thermal energy. Hence, the solar-to-electrical power conversion (P_{elec}) will have a negative sign in Eq. (1).

(2) It follows that when the solar array is *on*, a fraction (η_e) of the absorbed solar power (thermal energy) is converted to electrical energy. As such, the array surface temperature will be higher in the *off* condition than in the *on* condition.

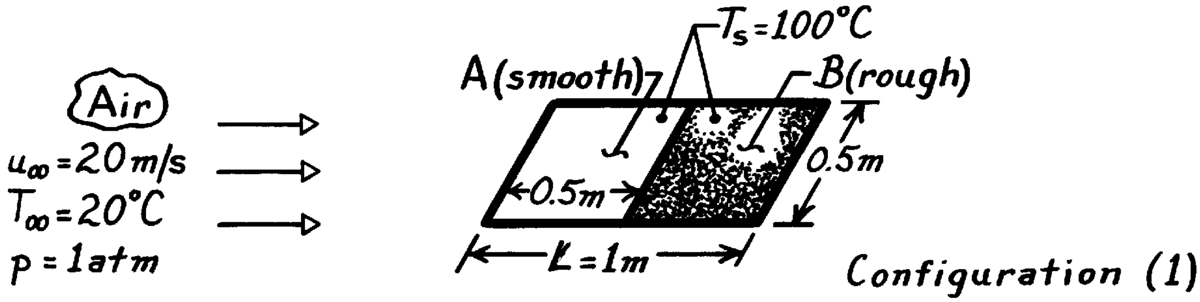
(3) Note that the assumed value for T_f at which to evaluate the properties was reasonable.

PROBLEM 7.21

KNOWN: Surface characteristics of a flat plate in an air stream.

FIND: Orientation which minimizes convection heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Surface B is sufficiently rough to trip the boundary layer when in the upstream position (Configuration 2).

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}$ m^2/s, k = 28.7×10^{-3} W/m·K, Pr = 0.7.

ANALYSIS: Since Configuration (2) results in a turbulent boundary layer over the entire surface, the lowest heat transfer is associated with Configuration (1). Find

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20 \text{ m/s} \times 1\text{m}}{19.2 \times 10^{-6} \text{m}^2/\text{s}} = 1.04 \times 10^6.$$

Hence in Configuration (1), transition will occur just before the rough surface (x_c = 0.48m). Note that

$$\overline{Nu}_{L,1} = \left[0.037\left(1.04 \times 10^6\right)^{4/5} - 871\right] 0.7^{1/3} = 1366$$

$$\overline{Nu}_{L,2} = 0.037\left(1.04 \times 10^6\right)^{4/5}(0.7)^{1/3} = 2139 > \overline{Nu}_{L,1}.$$

For Configuration (1): $$\frac{\bar{h}_{L,1} L}{k} = \overline{Nu}_{L,1} = 1366.$$

Hence

$$\bar{h}_{L,1} = 1366\left(28.7 \times 10^{-3} \text{ W/m} \cdot \text{K}\right)/1\text{m} = 39.2 \text{ W/m}^2 \cdot \text{K}$$

and

$$q_1 = \bar{h}_{L,1} A\left(T_s - T_\infty\right) = 39.2 \text{ W/m}^2 \cdot \text{K}\left(0.5\text{m} \times 1\text{m}\right)\left(100 - 20\right)\text{K}$$

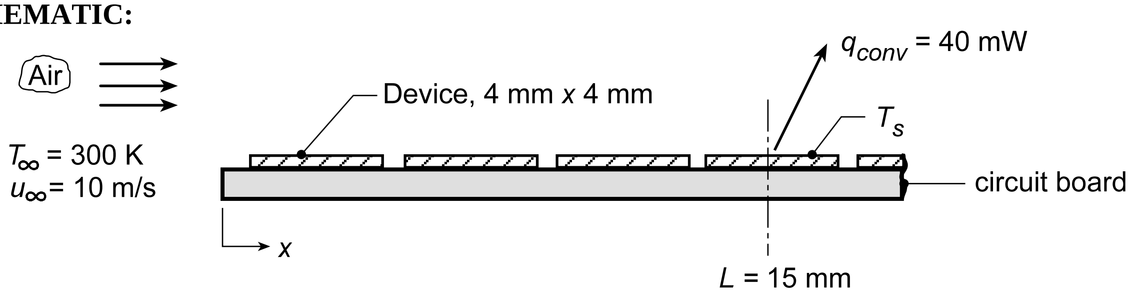
$$q_1 = 1568 \text{ W}.$$ <

PROBLEM 7.33

KNOWN: Air at 27°C with velocity of 10 m/s flows turbulently over a series of electronic devices, each having dimensions of 4 mm × 4 mm and dissipating 40 mW.

FIND: (a) Surface temperature T_s of the fourth device located 15 mm from the leading edge, (b) Compute and plot the surface temperatures of the first four devices for the range $5 \le u_\infty \le 15$ m/s, and (c) Minimum free stream velocity u_∞ if the surface temperature of the hottest device is not to exceed 80°C.

SCHEMATIC:



ASSUMPTIONS: (1) Turbulent flow, (2) Heat from devices leaving through top surface by convection only, (3) Device surface is isothermal, and (4) The average coefficient for the devices is equal to the local value at the mid position, i.e. $\bar{h}_4 = h_x(L)$.

PROPERTIES: *Table A.4*, Air (assume T_s = 330 K, $\bar{T} = (T_s + T_\infty)/2$ = 315 K, 1 atm): k = 0.0274 W/m·K, $\nu = 17.40 \times 10^{-6}$ m²/s, $\alpha = 24.7 \times 10^{-6}$ m²/s, Pr = 0.705.

ANALYSIS: (a) From Newton's law of cooling,

$$T_s = T_\infty + q_{conv}/\bar{h}_4 A_s \tag{1}$$

where $\bar{h}_4$ is the average heat transfer coefficient over the 4th device. Since flow is turbulent, it is reasonable and convenient to assume that

$$\bar{h}_4 = h_x(L = 15\,\text{mm}). \tag{2}$$

To estimate h_x, use the turbulent correlation evaluating thermophysical properties at $\bar{T}_f$ = 315 K (assume T_s = 330 K),

$$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3}$$

where

$$Re_x = \frac{u_\infty L}{\nu} = \frac{10\,\text{m/s} \times 0.015\,\text{m}}{17.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 8621$$

giving

$$Nu_x = \frac{h_x L}{k} = 0.0296(8621)^{4/5}(0.705)^{1/3} = 37.1$$

$$\bar{h}_4 = h_x = \frac{Nu_x k}{L} = \frac{37.1 \times 0.0274\,\text{W/m}\cdot\text{K}}{0.015\,\text{m}} = 67.8\,\text{W/m}^2\cdot\text{K}$$

Hence, with A_s = 4 mm × 4 mm, the surface temperature is

$$T_s = 300\,\text{K} + \frac{40 \times 10^{-3}\,\text{W}}{67.8\,\text{W/m}^2\cdot\text{K} \times \left(4 \times 10^{-3}\,\text{m}\right)^2} = 337\,\text{K} = 64^\circ\text{C}.$$ <

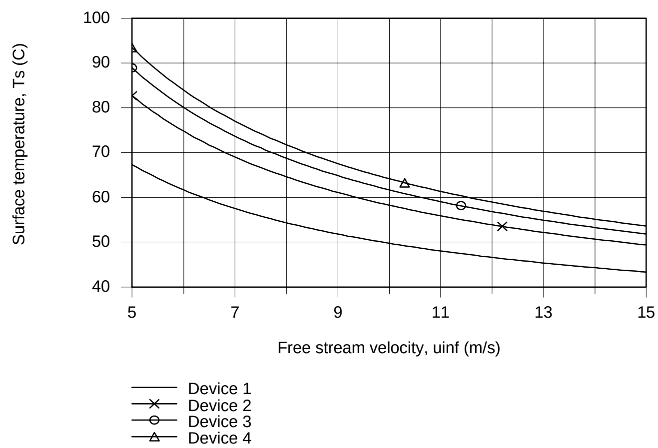
Continued...

PROBLEM 7.33 (Cont.)

(b) The surface temperature for each of the four devices (i = 1, 2, 3 4) follows from Eq. (1),

$$T_{s,i} = T_\infty + q_{conv} / \overline{h}_i A_s \qquad (3)$$

For devices 2, 3 and 4, $\overline{h}_i$ is evaluated as the local coefficient at the mid-positions, Eq. (2), x_2 = 6.5 mm, x_3 = 10.75 mm and x_4 = 15 mm. For device 1, $\overline{h}_1$ is the average value 0 to x_1, where evaluated $x_1 = L_1$ = 4.25 mm. Using Eq. (3) in the *IHT Workspace* along with the *Correlations Tool*, *External Flow*, *Local Coefficient* for *Laminar* or *Turbulent Flow*, the surface temperatures $T_{s,i}$ are determined as a function of the free stream velocity.



(c) Using the *Explore* option on the *Plot Window* associated with the IHT code of part (b), the minimum free stream velocity of

$$u_\infty = 6.6 \text{ m/s}$$ <

will maintain device 4, the hottest of the devices, at a temperature $T_{s,4}$ = 80°C.

COMMENTS: (1) Note that the thermophysical properties were evaluated at a reasonable assumed film temperature in part (a).

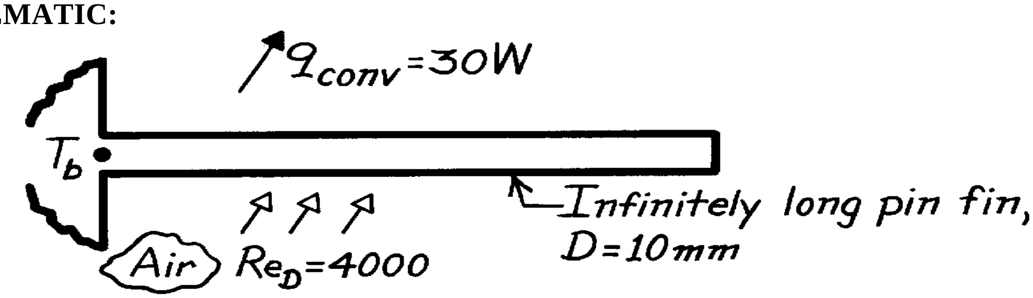
(2) From the $T_{s,i}$ vs. u_∞ plots, note that, as expected, the surface temperatures of the devices increase with distance from the leading edge.

PROBLEM 7.45

KNOWN: Pin fin of 10 mm diameter dissipates 30 W by forced convection in cross-flow of air with $Re_D = 4000$.

FIND: Fin heat rate if diameter is doubled while all conditions remain the same.

SCHEMATIC:



ASSUMPTIONS: (1) Pin behaves as infinitely long fin, (2) Conditions of flow, as well as base and air temperatures, remain the same for both situations, (3) Negligible radiation heat transfer.

ANALYSIS: For an infinitely long pin fin, the fin heat rate is

$$q_f = q_{conv} = \left(\overline{h} P k A_c\right)^{1/2} q_b$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence,

$$q_{conv} \sim \left(\overline{h} \cdot D \cdot D^2\right)^{1/2}.$$

For forced convection cross-flow over a cylinder, an appropriate correlation for estimating the dependence of $\overline{h}$ on the diameter is

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = C Re_D^m \, Pr^{1/3} = C\left(\frac{VD}{n}\right)^m Pr^{1/3}.$$

From Table 7.2 for $Re_D = 4000$, find m = 0.466 and

$$\overline{h} \sim D^{-1}\left(D\right)^{0.466} = D^{-0.534}.$$

It follows that

$$q_{conv} \sim \left(D^{-0.534} \cdot D \cdot D^2\right)^{1/2} = D^{1.23}.$$

Hence, with $q_1 \to D_1$ (10 mm) and $q_2 \to D_2$ (20 mm), find

$$q_2 = q_1\left(\frac{D_2}{D_1}\right)^{1.23} = 30\text{ W}\left(\frac{20}{10}\right)^{1.23} = 70.4\text{ W}.$$ <

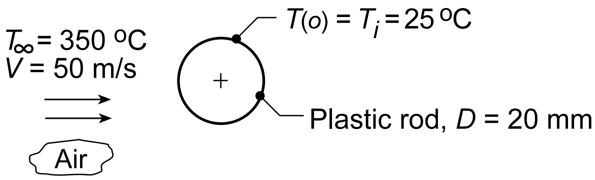
COMMENTS: The effect of doubling the diameter, with all other conditions remaining the same, is to increase the fin heat rate by a factor of 2.35. The effect is nearly linear, with enhancements due to the increase in surface and cross-sectional areas ($D^{1.5}$) exceeding the attenuation due to a decrease in the heat transfer coefficient ($D^{-0.267}$). Note that, with increasing Reynolds number, the exponent m increases and there is greater heat transfer enhancement due to increasing the diameter.

PROBLEM 7.62

KNOWN: Long coated plastic, 20-mm diameter rod, initially at a uniform temperature of T_i = 25°C, is suddenly exposed to the cross-flow of air at T_∞ = 350°C and V = 50 m/s.

FIND: (a) Time for the surface of the rod to reach 175°C, the temperature above which the special coating cures, and (b) Compute and plot the time-to-reach 175°C as a function of air velocity for 5 ≤ V ≤ 50 m/s.

SCHEMATIC:



ASSUMPTIONS: (a) One-dimensional, transient conduction in the rod, (2) Constant properties, and (3) Evaluate thermophysical properties at $T_f = [(T_s + T_i)/2 + T_\infty] = [(175 + 25)/2 + 350]$°C = 225°C = 500 K.

PROPERTIES: Rod (Given): ρ = 2200 kg/m^3, c = 800 J/kg·K, k = 1 W/m·K, α = k/ρc = 5.68 × 10^{-7} m^2/s; *Table A.4*, Air (T_f ≈ 500 K, 1 atm): ν = 38.79 × 10^{-6} m^2/s, k = 0.0407 W/m·K, Pr = 0.684.

ANALYSIS: (a) To determine whether the lumped capacitance method is valid, determine the Biot number

$$Bi_{lc} = \frac{\bar{h}(r_o/2)}{k} \qquad (1)$$

The convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.63 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = 50\,m/s \times 0.020\,m \Big/ 38.79\times10^{-6}\,m^2/s = 25,780$$

$$\bar{h} = \frac{0.0407\,W/m\cdot K}{0.020\,m}\left\{0.3 + \frac{0.63(25,780)^{1/2}(0.684)^{1/3}}{\left[1+(0.4/0.684)^{2/3}\right]^{1/4}}\left[1+\left(\frac{25,780}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 184\ W/m^2\cdot K \quad (2)$$

Substituting for $\bar{h}$ from Eq. (2) into Eq. (1), find

$$Bi_{lc} = 184\,W/m^2\cdot K\,(0.010\,m/2)/1\,W/m\cdot K = 0.92 \qquad >> 0.1$$

Hence, the lumped capacitance method is inappropriate. Using the one-term series approximation, Section 5.6.2, Eqs. 5.49 with Table 5.1,

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) J_o\left(\zeta_1 r^*\right) \qquad r^* = r/r_o = 1$$

$$\theta^* = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = \frac{(175-350)^\circ C}{(25-350)^\circ C} = 0.54$$

$$Bi = \bar{h}r_o/k = 1.84 \qquad \zeta_1 = 1.5308\ rad \qquad C_1 = 1.3384$$

Continued...

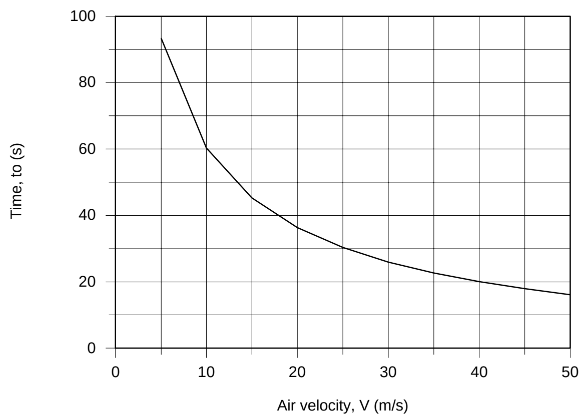
$$0.54 = 1.3384\exp[-(1.5308\text{rad})^2 Fo] J_o(1.5308 \times 1)$$

Using Table B.4 to evaluate $J_o(1.5308) = 0.4944$, find Fo = 0.0863 where

$$Fo = \frac{\alpha t_o}{r_o^2} = \frac{5.68 \times 10^{-7}\,\text{m}^2/\text{s} \times t_o}{(0.010\,\text{m})^2} = 5.68 \times 10^{-3} t_o \qquad (6)$$

$$t_o = 15.2\text{s}$$ <

(b) Using the *IHT Model*, *Transient Conduction*, *Cylinder*, and the *Tool*, *Correlations*, *External Flow*, *Cylinder*, results for the time-to-reach a surface temperature of 175°C as a function of air velocity V are plotted below.



COMMENTS: (1) Using the *IHT Tool*, *Correlations*, *External Flow*, *Cylinder*, the effect of the film temperature T_f on the estimated convection coefficient with V = 50 m/s can be readily evaluated.

T_f (K)	460	500	623
$\bar{h}$ ($W/m^2 \cdot K$)	187	184	176

At early times, $\bar{h}$ = 184 $W/m^2 \cdot K$ is a good estimate, while as the cylinder temperature approaches the airsteam temperature, the effect starts to be noticeable (10% decrease).

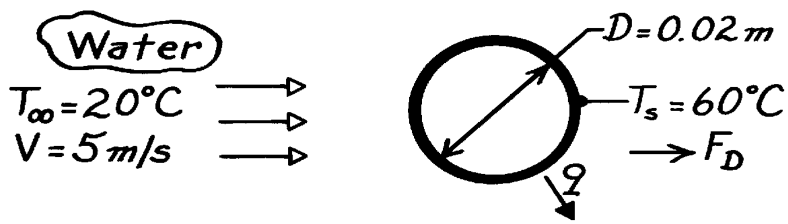
(2) The IHT analysis performed for part (b) was developed in two parts. Using a known value for $\bar{h}$, the *Transient Conduction*, *Cylinder Model* was tested. Separately, the *Correlation Tools* was assembled and tested. Then, the two files were merged to give the workspace for determining the time-to-reach 175°C as a function of velocity V.

PROBLEM 7.65

KNOWN: Temperature and velocity of water flowing over a sphere of prescribed temperature and diameter.

FIND: (a) Drag force, (b) Rate of heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-6*, Saturated Water (T_∞ = 293K): ρ = 998 kg/m^3, $\mu = 1007 \times 10^{-6}$ N·s/m^2, k = 0.603 W/m·K, Pr = 7.00; (T_s = 333 K): $\mu = 467 \times 10^{-6}$ N·s/m^2; (T_f = 313 K): ρ = 992 kg/m^3, μ = 657×10^{-6} N·s/m^2.

ANALYSIS: (a) Evaluating μ and ρ at the film temperature,

$$Re_D = \frac{\rho VD}{\mu} = \frac{\left(992\ \text{kg/m}^3\right)5\ \text{m/s}\ (0.02\ \text{m})}{657\times10^{-6}\ \text{N}\cdot\text{s/m}^2} = 1.51\times10^5$$

and from Fig. 7.8, C_D = 0.42. Hence

$$F_D = C_D \frac{\pi D^2}{4} \rho \frac{V^2}{2} = 0.42 \frac{\pi\ (0.02\ \text{m})^2}{4} 992 \frac{\text{kg}}{\text{m}^3} \frac{(5\ \text{m/s})^2}{2} = 1.64\ \text{N}. \qquad <$$

(b) With the Reynolds number evaluated at the free stream temperature,

$$Re_D = \frac{\rho VD}{\mu} = \frac{998\ \text{kg/m}^3 (5\ \text{m/s})\ (0.02\ \text{m})}{1007\times10^{-6}\ \text{N}\cdot\text{s/m}^2} = 9.91\times10^4$$

it follows from the Whitaker relation that

$$\overline{Nu}_D = 2 + \left[0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right]Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4\left(9.91\times10^4\right)^{1/2} + 0.06\left(9.91\times10^4\right)^{2/3}\right](7.0)^{0.4}\left(\frac{1007}{467}\right)^{1/4} = 673.$$

Hence, the convection coefficient and heat rate are

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.603\ \text{W/m}\cdot\text{K}}{0.02\ \text{m}} 673 = 20{,}300\ \text{W/m}^2\cdot\text{K}$$

$$q = \bar{h}\left(\pi D^2\right)\ (T_s - T_\infty) = 20{,}300 \frac{\text{W}}{\text{m}^2\cdot\text{K}} \pi\ (0.02\ \text{m})^2\ (60-20)^\circ\text{C} = 1020\ \text{W}. \qquad <$$

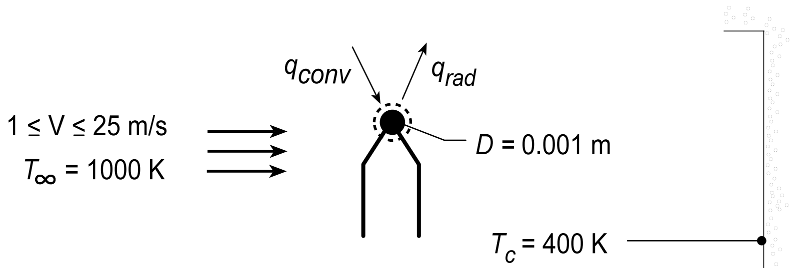
COMMENTS: Compare the foregoing value of $\bar{h}$ with that obtained in the text example under similar conditions. The significant increase in $\bar{h}$ is due to the much larger value of k and smaller value of ν for the water. Note that Re_D is slightly beyond the range of the correlation.

PROBLEM 7.78

KNOWN: Velocity and temperature of combustion gases. Diameter and emissivity of thermocouple junction. Combustor temperature.

FIND: (a) Time to achieve 98% of maximum thermocouple temperature rise, (b) Steady-state thermocouple temperature, (c) Effect of gas velocity and thermocouple emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Constant properties, (3) Negligible conduction through lead wires, (4) Radiation exchange between small surface and a large enclosure (parts b and c).

PROPERTIES: Thermocouple (given): $0.1 \le \varepsilon \le 1.0$, k = 100 W/m·K, c = 385 J/kg·K, ρ = 8920 kg/m^3; Gases (given): k = 0.05 W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, Pr = 0.69.

ANALYSIS: (a) If the lumped capacitance analysis may be used, it follows from Equation 5.5 that

$$t = \frac{\rho Vc}{\bar{h}A_s}\ln\frac{T_i - T_\infty}{T - T_\infty} = \frac{D\rho c}{6\bar{h}}\ln(50).$$

Neglecting the viscosity ratio correlation for variable property effects, use of V = 5 m/s with the Whitaker correlation yields

$$\overline{Nu}_D = (\bar{h}D/k) = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4} \qquad Re_D = \frac{VD}{\nu} = \frac{5\,m/s(0.001\,m)}{50\times10^{-6}\,m^2/s} = 100$$

$$\bar{h} = \frac{0.05\,W/m\cdot K}{0.001\,m}\left[2 + \left(0.4(100)^{1/2} + 0.06(100)^{2/3}\right)(0.69)^{0.4}\right] = 328\,W/m^2\cdot K$$

Since Bi = $\bar{h}(r_o/3)/k$ = 5.5×10^{-4}, the lumped capacitance method may be used. Hence,

$$t = \frac{0.001\,m\left(8920\,kg/m^3\right)385\,J/kg\cdot K}{6\times328\,W/m^2\cdot K}\ln(50) = 6.83s \qquad <$$

(b) Performing an energy balance on the junction and evaluating radiation exchange from Equation 1.7, $q_{conv} = q_{rad}$. Hence, with $\varepsilon = 0.5$,

$$\bar{h}A_s(T_\infty - T) = \varepsilon A_s\sigma\left(T^4 - T_c^4\right)$$

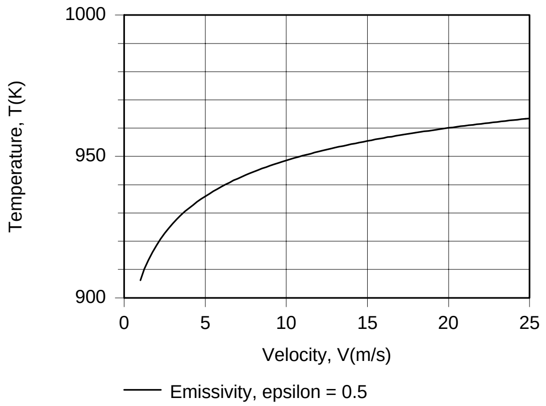
$$(1000 - T)K = \frac{0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4}{328\,W/m^2\cdot K}\left[T^4 - (400)^4\right]K^4.$$

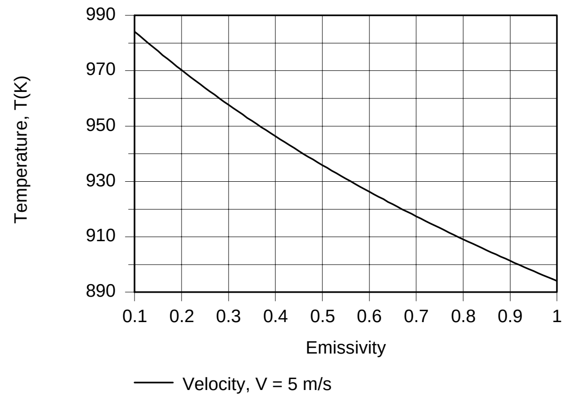
$$T = 936\,K \qquad <$$

(c) Using the *IHT First Law Model* for a *Solid Sphere* with the appropriate *Correlation* for external flow from the Tool Pad, parametric calculations were performed to determine the effects of V and ε_g, and the following results were obtained.

Continued...

PROBLEM 7.78 (Cont.)





Since the temperature recorded by the thermocouple junction increases with increasing V and decreasing ε, the measurement error, T_∞ - T, decreases with increasing V and decreasing ε. The error is due to net radiative transfer from the junction (which depresses T) and hence should decrease with decreasing ε. For a prescribed heat loss, the temperature difference (T_∞ - T) decreases with decreasing convection resistance, and hence with increasing h(V).

COMMENTS: To infer the actual gas temperature (1000 K) from the measured result (936 K), correction would have to be made for radiation exchange with the cold surroundings.