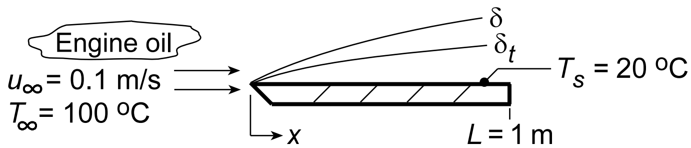
PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for $0 \le x \le 1$ m.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): $\rho = 864\ \text{kg/m}^3$, $\nu = 86.1 \times 10^{-6}\ \text{m}^2\text{/s}$, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{0.1\,\text{m/s} \times 1\,\text{m}}{86.1 \times 10^{-6}\,\text{m}^2/\text{s}} = 1161$$

Hence the flow is laminar at x = L, from Eqs. 7.19 and 7.24, and

$$\delta = 5L\,\text{Re}_L^{-1/2} = 5(1\,\text{m})(1161)^{-1/2} = 0.147\,\text{m} \qquad <$$

$$\delta_t = \delta\,\text{Pr}^{-1/3} = 0.147\,\text{m}(1081)^{-1/3} = 0.0143\,\text{m} \qquad <$$

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3} = \frac{0.140\,\text{W/m}\cdot\text{K}}{1\text{m}} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\,\text{W/m}^2\cdot\text{K}$$

$$q''_x = h_L(T_s - T_\infty) = 16.25\,\text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -1300\,\text{W/m}^2 \qquad <$$

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,\text{Re}_L^{-1/2} = \frac{864\,\text{kg/m}^3}{2}(0.1\,\text{m/s})^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,\text{kg/m}\cdot\text{s}^2 = 0.0842\,\text{N/m}^2 \qquad <$$

(c) With the drag force per unit width given by $D' = 2L\bar{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,\text{Re}_L^{-1/2} = (1\,\text{m})864\,\text{kg/m}^3\,(0.1\,\text{m/s})^2 / 2\ 1.328(1161)^{-1/2} = 0.337\,\text{N/m} \qquad <$$

For laminar flow, the average value $\bar{h}_L$ over the distance 0 to L is twice the local value, h_L,

$$\bar{h}_L = 2h_L = 32.5\,\text{W/m}^2\cdot\text{K}$$

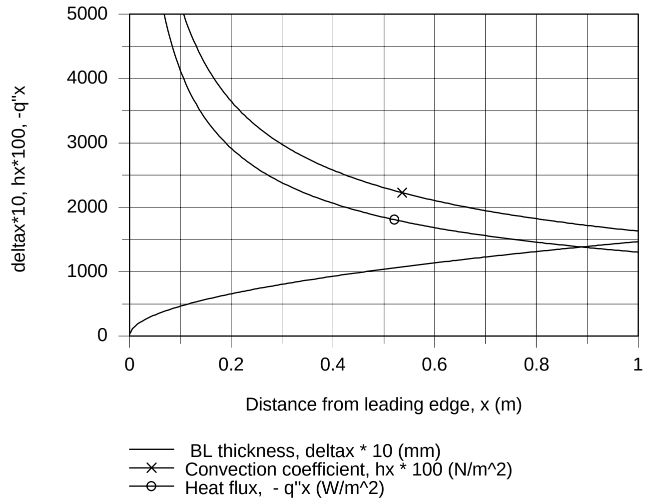
The total heat transfer rate per unit width of the plate is

$$q' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\,\text{m})32.5\,\text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -5200\,\text{W/m} \qquad <$$

Continued...

PROBLEM 7.2 (Cont.)

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.



COMMENTS: (1) Note that since Pr >> 1, $\delta \gg \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

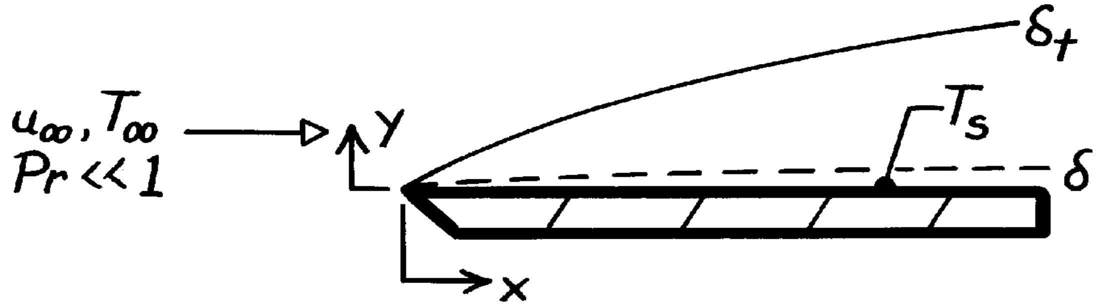
```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q''x
q''x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                  // Scaling parameter for convenience in plotting
q''x_plot = ( -1 ) * q''x           // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)            // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)              // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)              // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                    // Film temperature, K
Tinf = 100 + 273                        // Freestream temperature, K
Ts = 20 + 273                           // Surface temperature, K
uinf = 0.1                              // Freestream velocity, m/s
x = 1                                   // Plate length, m
```

PROBLEM 7.4

KNOWN: Liquid metal in parallel flow over a flat plate.

FIND: An expression for the local Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, (2) $\delta << \delta_t$, hence $u(y) \approx u_\infty$, (3) Boundary layer approximations are valid, (4) Constant properties.

ANALYSIS: The boundary layer energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}.$$

Assuming $u(y) = u_\infty$, it follows that $v = 0$ and the energy equation becomes

$$u_\infty\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{or} \qquad \frac{\partial T}{\partial x} = \frac{\alpha}{u_\infty}\frac{\partial^2 T}{\partial y^2}.$$

Boundary Conditions: $T(x,0) = T_s$, $T(x,\infty) = T_\infty$.

Initial Condition: $T(0,y) = T_\infty$.

The differential equation is analogous to that for transient one-dimensional conduction in a plane wall, and the conditions are analogous to those of Fig. 5.7, Case (1). Hence the solution is given by Eqs. 5.57 and 5.58. Substituting y for x, x for t, T_∞ for T_i, and α/u_∞ for α, the boundary layer temperature and the surface heat flux become

$$\frac{T(x,y) - T_s}{T_\infty - T_s} = \operatorname{erf}\left[\frac{y}{2\left(\alpha x/u_\infty\right)^{1/2}}\right]$$

$$q_s'' = \frac{k\left(T_s - T_\infty\right)}{\left(\pi \alpha x/u_\infty\right)^{1/2}}.$$

Hence, with
$$Nu_x \equiv \frac{hx}{k} = \frac{q_s'' x}{\left(T_s - T_\infty\right)k}$$

find
$$Nu_x = \frac{x}{\left(\pi \alpha x/u_\infty\right)^{1/2}} = \frac{\left(xu_\infty\right)^{1/2}}{\pi^{1/2}\left(k/\rho c_p\right)^{1/2}} = \frac{1}{\pi^{1/2}}\left[\frac{\rho u_\infty x}{\mu}\cdot\frac{c_p\mu}{k}\right]^{1/2}$$

$$Nu_x = 0.564\left(Re_x Pr\right)^{1/2} = 0.564\, Pe^{1/2}$$ <

where Pe = Re · Pr is the Peclet number.

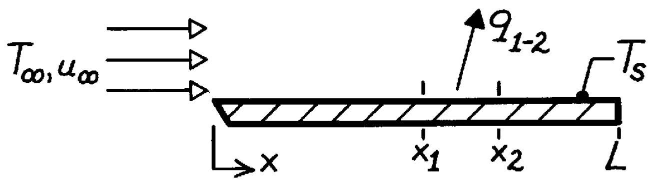
COMMENTS: Because k is very large, axial conduction effects may not be negligible. That is, the $\alpha\, \partial^2 T/\partial x^2$ term of the energy equation may be important.

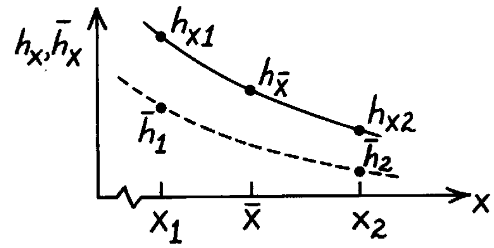
PROBLEM 7.7

KNOWN: Parallel flow over a flat plate and two locations representing a short span x_1 to x_2 where $(x_2 - x_1) << L$.

FIND: Three different expressions for the average heat transfer coefficient over the short span x_1 to x_2, $\bar{h}_{1-2}$.

SCHEMATIC:





ASSUMPTIONS: (1) Parallel flow over a flat plate.

ANALYSIS: The heat rate per unit width for the span can be written as

$$q'_{1-2} = \bar{h}_{1-2}\left(x_2 - x_1\right)\left(T_s - T_\infty\right) \tag{1}$$

where $\bar{h}_{1-2}$ is the average heat transfer coefficient over the span and can be evaluated in either of the following three ways:

(a) *Local coefficient at* $\bar{x} = \left(x_1 + x_2\right)/2$. If the span is very short, it is reasonable to assume that

$$\bar{h}_{1-2} \approx h_{\bar{x}} \tag{2}$$

where $h_{\bar{x}}$ is the local convection coefficient at the mid-point of the span.

(b) *Local coefficients at x_1 and x_2*. If the span is very short it is reasonable to assume that $\bar{h}_{1-2}$ is the average of the local values at the ends of the span,

$$\bar{h}_{1-2} \approx \left[h_{x1} + h_{x2}\right]/2. \tag{3}$$

(c) *Average coefficients for x_1 and x_2*. The heat rate for the span can also be written as

$$q'_{1-2} = q'_{0-2} - q'_{0-1} \tag{4}$$

where the rate q_{0-x} denotes the heat rate for the plate over the distance from 0 to x. In terms of heat transfer coefficients, find

$$\bar{h}_{1-2} \cdot \left(x_2 - x_1\right) = \bar{h}_2 \cdot x_2 - \bar{h}_1 \cdot x_1$$

$$\bar{h}_{1-2} = \bar{h}_2 \frac{x_2}{x_2 - x_1} - \bar{h}_1 \frac{x_1}{x_2 - x_1} \tag{5}$$

where $\bar{h}_1$ and $\bar{h}_2$ are the average coefficients from 0 to x_1 and x_2, respectively.

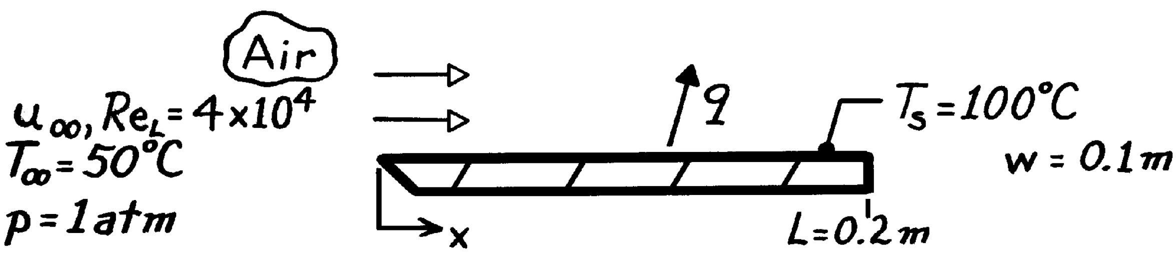
COMMENTS: Eqs. (2) and (3) are approximate and work better when the span is small and the flow is turbulent rather than laminar ($h_x \sim x^{-0.2}$ vs $h_x \sim x^{-0.5}$). Of course, we require that $x_c < x_1$, x_2 or $x_c > x_1$, x_2; that is, the approximations are inappropriate around the transition region. Eq. (5) is an exact relationship, which applies under any conditions.

PROBLEM 7.15

KNOWN: Temperature, pressure and Reynolds number for air flow over a flat plate of uniform surface temperature.

FIND: (a) Rate of heat transfer from the plate, (b) Rate of heat transfer if air velocity is doubled and pressure is increased to 10 atm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L (w\times L)\ (T_s - T_\infty).$$

Since the flow is laminar over the entire plate for $Re_L = 4\times10^4$, it follows that

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(40{,}000)^{1/2}(0.70)^{1/3} = 117.9.$$

Hence $$\bar{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ W/m\cdot K}{0.2m} = 17.6\ W/m^2\cdot K$$

and $$q = 17.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 17.6\ W.$$ <

(b) With $p_2 = 10\ p_1$, it follows that $\rho_2 = 10\ \rho_1$ and $\nu_2 = \nu_1/10$. Hence

$$Re_{L,2} = \left(\frac{u_\infty L}{n}\right)_2 = 2\times10\left(\frac{u_\infty L}{n}\right)_1 = 20\ Re_{L,1} = 8\times10^5$$

and mixed boundary layer conditions exist on the plate. Hence

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left(0.037\ Re_L^{4/5} - 871\right)\ Pr^{1/3} = \left[0.037\times\left(8\times10^5\right)^{4/5} - 871\right]\ (0.70)^{1/3}$$

$$\overline{Nu}_L = 961.$$

Hence, $$\bar{h}_L = 961\frac{0.0299\ W/m\cdot K}{0.2m} = 143.6\ W/m^2\cdot K$$

$$q = 143.6\frac{W}{m^2\cdot K}(0.1m\times0.2m)\ (100-50)^\circ C = 143.6\ W.$$ <

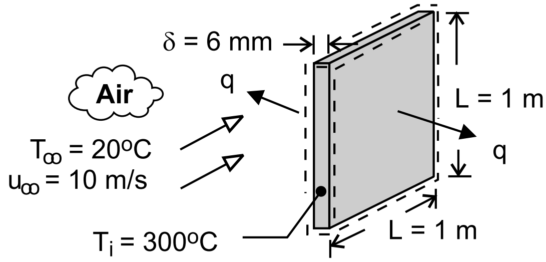
COMMENTS: Note that, in calculating $Re_{L,2}$, ideal gas behavior has been assumed. It has also been assumed that k, μ and Pr are independent of pressure over the range considered.

PROBLEM 7.25

KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from sides of plate, (5) $Re_{x,c} = 5\times10^5$, (6) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h}A_s(T_i - T_\infty) = 2\bar{h}L^2(T_i - T_\infty)$$

With $Re_L = u_\infty L/\nu = 10\,m/s\times1m/30.4\times10^{-6}m^2/s = 3.29\times10^5$, flow is laminar over the entire surface and

$$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3} = 0.664\left(3.29\times10^5\right)^{1/2}(0.688)^{1/3} = 336$$

$$\bar{h} = (k/L)\overline{Nu}_L = (0.0361\,W/m\cdot K/1m)336 = 12.1\,W/m^2\cdot K$$

Hence,

$$q = 2\times12.1\,W/m^2\cdot K(1m)^2(300-20)^\circ C = 6780\,W$$ <

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain (Eq. 5.2),

$$\rho\delta L^2 c\left.\frac{dT}{dt}\right|_i = -\bar{h}2L^2(T_i - T_\infty)$$

$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,W/m^2\cdot K\right)(300-20)^\circ C}{7832\,kg/m^3\times0.006m\times549\,J/kg\cdot K} = -0.26^\circ C/s$$ <

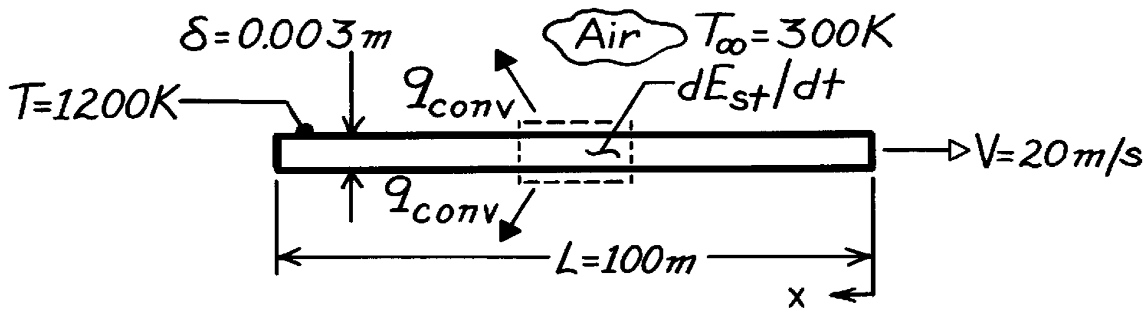
COMMENTS: (1) With $Bi = \bar{h}(\delta/2)/k_p = 7.4\times10^{-4}$, use of the lumped capacitance method is appropriate. (2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

PROBLEM 7.28

KNOWN: Length, thickness, speed and temperature of steel strip.

FIND: Rate of change of strip temperature 1 m from leading edge and at trailing edge. Location of minimum cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is 5×10^5.

PROPERTIES: Steel (given): ρ = 7900 kg/m^3, c_p = 640 J/kg·K. *Table A-4*, Air $\left(\overline{T} = 750\text{K, 1 atm}\right)$: $\nu = 76.4 \times 10^{-6}$ m^2/s, k = 0.0549 W/m·K, Pr = 0.702.

ANALYSIS: Performing an energy balance for a control mass of unit surface area A_s riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s \left(T - T_\infty\right) = r d A_s c_p \left(dT/dt\right)$$

$$dT/dt) = \frac{-2h_x\left(T - T_\infty\right)}{r d c_p} = -\frac{2\left(900\text{K}\right)h_x}{7900\ \text{kg/m}^3\left(0.003\ \text{m}\right)640\ \text{J/kg}\cdot\text{K}} = -0.119 h_x\ \left(\text{K/s}\right).$$

At x = 1 m, $\quad \text{Re}_x = \frac{Vx}{n} = \frac{20\ \text{m/s}\left(1\text{m}\right)}{76.4\times10^{-6}\ \text{m}^2/\text{s}} = 2.62\times10^5 < \text{Re}_{x,c}$. Hence,

$$h_x = \left(k/x\right)0.332\text{Re}_x^{1/2}\text{Pr}^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{1\ \text{m}}\left(0.332\right)\left(2.62\times10^5\right)^{1/2}\left(0.702\right)^{1/3} = 8.29\text{W/m}^2\cdot\text{K}$$

and at x = 1m, $\quad$ dT/dt) = -0.987 K/s. $\quad$ <

At the trailing edge, $\text{Re}_x = 2.62\times10^7 > \text{Re}_{x,c}$. Hence

$$h_x = \left(k/x\right)0.0296\text{Re}_x^{4/5}\text{Pr}^{1/3} = \frac{0.0549\ \text{W/m}\cdot\text{K}}{100\ \text{m}}\left(0.0296\right)\left(2.62\times10^7\right)^{4/5}\left(0.702\right)^{1/3} = 12.4\text{W/m}^2\cdot\text{K}$$

and at x = 100 m, $\quad$ dT/dt) = -1.47 K/s. $\quad$ <

The minimum cooling rate occurs just before transition; hence, for $\text{Re}_{x,c} = 5\times10^5$

$$x_c = 5\times10^5\left(n/V\right) = \frac{5\times10^5\times76.4\times10^{-6}\text{m}^2/\text{s}}{20\ \text{m/s}} = 1.91\ \text{m}$$ <

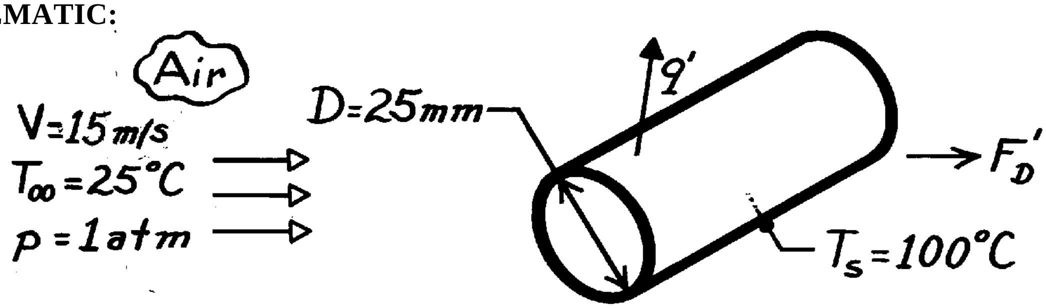
COMMENTS: The cooling rates are very low and would remain low even if radiation were considered. For this reason, hot strip metals are quenched by water and not by air.

PROBLEM 7.42

KNOWN: Conditions associated with air in cross flow over a pipe.

FIND: (a) Drag force per unit length of pipe, (b) Heat transfer per unit length of pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature, (3) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = 335 K, 1 atm): ν = 19.31 × 10^{-6} m^2/s, ρ = 1.048 kg/m^3, k = 0.0288 W/m·K, Pr = 0.702.

ANALYSIS: (a) From the definition of the drag coefficient with A_f = DL, find

$$F_D = C_D A_f \frac{rV^2}{2}$$

$$F_D' = C_D D \frac{rV^2}{2}.$$

With

$$Re_D = \frac{VD}{n} = \frac{15 \text{ m/s} \times (0.025 \text{ m})}{19.31 \times 10^{-6} \text{ m}^2/\text{s}} = 1.942 \times 10^4$$

from Fig. 7.8, $C_D \approx 1.1$. Hence

$$F_D = 1.1(0.025 \text{ m})\ 1.048 \text{ kg/m}^3\ (15 \text{ m/s})^2/2 = 3.24 \text{ N/m}. \qquad <$$

(b) Using Hilpert's relation, with C = 0.193 and m = 0.618 from Table 7.2,

$$\overline{h} = \frac{k}{D} C\, Re_D^m\ Pr^{1/3} = \frac{0.0288 \text{ W/m}\cdot\text{K}}{0.025 \text{ m}} \times 0.193\left(1.942 \times 10^4\right)^{0.618} (0.702)^{1/3}$$

$$\overline{h} = 88 \text{ W/m}^2\cdot\text{K}.$$

Hence, the heat rate per unit length is

$$q' = \overline{h}\,(pD)\ (T_s - T_\infty) = 88 \text{ W/m}^2\cdot\text{K}\,(p \times 0.025 \text{ m})\ (100-25)^\circ\text{C} = 520 \text{ W/m}. \qquad <$$

COMMENTS: Using the Zhukauskas correlation and evaluating properties at T_∞ (ν = 15.71 × 10^{-6} m^2/s, k = 0.0261 W/m·K, Pr = 0.707), but with Pr_s = 0.695 at T_s,

$$\overline{h} = \frac{0.0261}{0.025} 0.26 \left(\frac{15 \times 0.025}{15.71 \times 10^{-6}}\right)^{0.6} (0.707)^{0.37} (0.707/0.695)^{1/4} = 102 \text{ W/m}^2\cdot\text{K}.$$

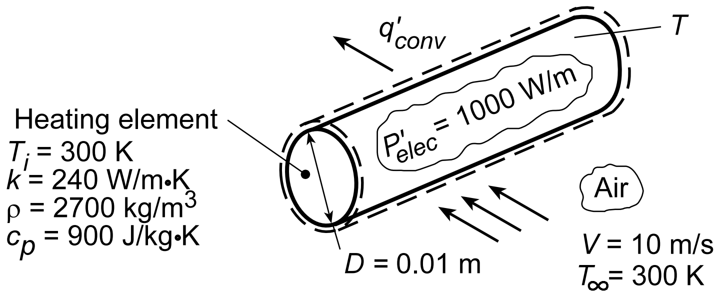
This result agrees with that obtained from Hilpert's relation to within the uncertainty normally associated with convection correlations.

PROBLEM 7.43

KNOWN: Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform heater temperature, (2) Negligible radiation.

PROPERTIES: *Table A.4,* air (assume $T_f \approx 450$ K): $\nu = 32.39 \times 10^{-6}$ m^2/s, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} = \bar{h}(\pi D)(T - T_\infty) = P'_{elec} = 1000\,\text{W/m}$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(10\,\text{m/s})0.01\,\text{m}}{32.39\times10^{-6}\,\text{m}^2/\text{s}} = 3{,}087$$

the Churchill and Bernstein correlation, Eq. 7.57, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(3087)^{1/2}(0.686)^{1/3}}{\left[1+(0.4/0.686)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3087}{282{,}000}\right)^{5/8}\right]^{4/5} = 28.2$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0373\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}28.2 = 105.2\,\text{W/m}^2\cdot\text{K}$$

Hence, the steady-state temperature is

$$T = T_\infty + \frac{P'_{elec}}{\pi D\bar{h}} = 300\,\text{K} + \frac{1000\,\text{W/m}}{\pi(0.01\,\text{m})105.2\,\text{W/m}^2\cdot\text{K}} = 603\,\text{K} \quad <$$

(b) With Bi = $\bar{h}r_o/k$ = 105.2 W/m^2·K(0.005 m)/240 W/m·K = 0.0022, a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for $T_i = T_\infty$, reduces to

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

Continued...

where a = $4\bar{h}/D\rho c_p = (4 \times 105.2\,\text{W/m}^2\cdot\text{K})/(0.01\,\text{m} \times 2700\,\text{kg/m}^3 \times 900\,\text{J/kg}\cdot\text{K})$ = 0.0173 s^{-1} and b/a = $P'_{elec}/\pi D\bar{h} = 1000\,\text{W/m}/\pi(0.01\,\text{m} \times 105.2\,\text{W/m}^2\cdot\text{K})$ = 302.6 K. Hence,

$$\left[1 - \exp(-0.0173t)\right] = \frac{(593 - 300)\,\text{K}}{302.6\,\text{K}} = 0.968$$

$t \approx 200\text{s}$ <

COMMENTS: (1) For T = 603 K and a representative emissivity of ε = 0.8, net radiation exchange between the heater and surroundings at $T_{sur} = T_\infty$ = 300 K would be $q'_{rad} = \varepsilon\sigma(\pi D)(T^4 - T_{sur}^4)$ = 0.8 × 5.67 × 10^{-8} W/m$^2\cdot$K^4 (π × 0.01 m)(603^4 - 300^4)K^4 = 177 W/m. Hence, although small, radiation exchange is not negligible. The effects of radiation are considered in Problem 7.46.

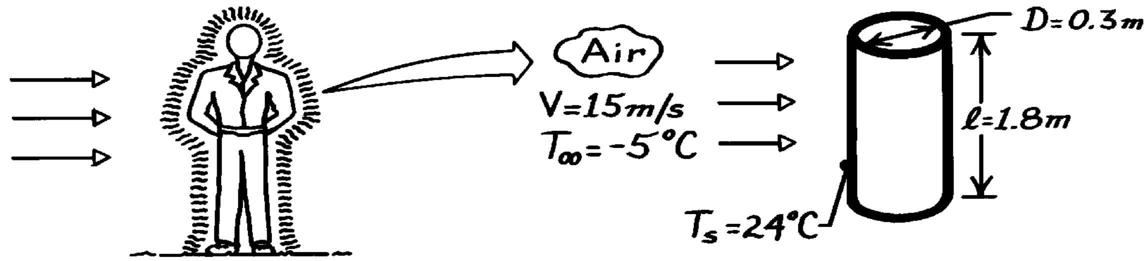
(2) The assumed value of T_f is very close to the actual value, rendering the selected air properties accurate.

PROBLEM 7.57

KNOWN: Person, approximated as a cylinder, is subjected to prescribed convection conditions.

FIND: Heat rate from body for prescribed temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Person can be approximated by cylindrical form having uniform surface temperature, (3) Negligible heat loss from cylinder top and bottom surfaces, (4) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_∞ = 268 K, 1 atm): $\nu = 13.04 \times 10^{-6}$ m^2/s, $k = 23.74 \times 10^{-3}$ W/m·K, Pr = 0.725; (T_s = 297 K, 1 atm): Pr = 0.707.

ANALYSIS: The heat transfer rate from the cylinder, approximating the person, is given as

$$q = \overline{h} A_s \left(T_s - T_\infty \right)$$

where $A_s = p D \ell$ and $\overline{h}$ must be estimated from a correlation appropriate to cross-flow over a cylinder. Use the Zhukauskas relation,

$$\overline{Nu}_D = \frac{\overline{h} D}{k} = C \, Re_D^m \, Pr^n \left(Pr/Pr_s \right)^{1/4}$$

and calculate the Reynold's number,

$$Re_D = \frac{VD}{n} = \frac{15 \text{m/s} \times 0.3 \text{ m}}{13.04 \times 10^{-6} \text{ m}^2 / \text{s}} = 345{,}092.$$

From Table 7-4, find C = 0.076 and m = 0.7. Since Pr < 10, n = 0.37, giving

$$\overline{Nu}_D = 0.076 \; (345{,}092)^{0.7} \, 0.725^{0.37} \left(\frac{0.725}{0.707} \right)^{1/4} = 511$$

$$\overline{h} = \overline{Nu}_D \frac{k}{D} = \frac{511 \times 23.74 \times 10^{-3} \text{W/m} \cdot \text{K}}{0.3 \text{ m}} = 40.4 \text{ W/m}^2 \cdot \text{K}.$$

The heat transfer rate is

$$q = 40.4 \text{W/m}^2 \cdot \text{K} \left(p \times 0.3 \text{ m} \times 1.8 \text{ m} \right) \; \left(24 - (-5) \right)^\circ \text{C} = 1988 \text{ W}. \qquad <$$

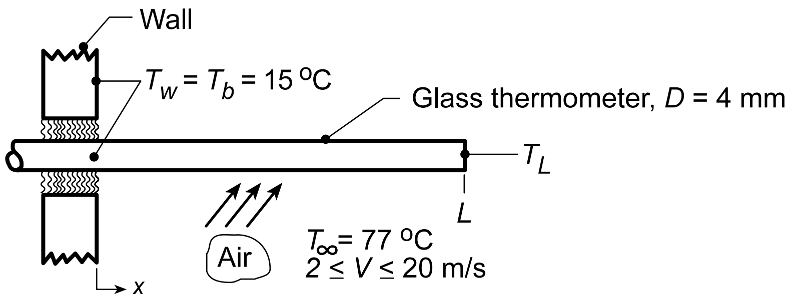
COMMENTS: Note the temperatures at which properties are evaluated for the Zhukauskas correlation.

PROBLEM 7.59

KNOWN: Mercury-in-glass thermometer mounted on duct wall used to measure air temperature.

FIND: (a) Relationship for the immersion error, $\Delta T_i = T(L) - T_\infty$ as a function of air velocity, thermometer diameter and length, (b) Length of insertion if ΔT_i is not to exceed 0.25°C when the air velocity is 10 m/s, (c) For the length of part (b), calculate and plot ΔT_i as a function of air velocity for 2 to 20 m/s, and (d) For a given insertion length, will ΔT_i increase or decrease with thermometer diameter increase; is ΔT_i more sensitive to diameter or velocity changes?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Thermometer approximates a one-dimensional (glass) fin with an *adiabatic* tip, (3) Convection coefficient is uniform over length of thermometer.

PROPERTIES: *Table A.3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A.4*, Air (T_f = (15 + 77)°C/2 ≈ 320 K, 1 atm): k = 0.0278 W/m·K, $\nu = 17.90 \times 10^{-6}$ m/s^2, Pr = 0.704.

ANALYSIS: (a) From the analysis of a one-dimensional fin, see Table 3.4,

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(mL)} \qquad m^2 = \frac{\bar{h}P}{k_g A_c} = \frac{4\bar{h}}{k_g D} \tag{1}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence, the immersion error is

$$\Delta T_i = T(L) - T_\infty = (T_b - T_\infty)/\cosh(mL). \tag{2}$$

Using the Hilpert correlation for the circular cylinder in cross flow,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^{1/3} = \frac{k}{D} C \left(\frac{VD}{\nu}\right)^m Pr^{1/3} = \frac{k\, Pr^{1/3}}{\nu^m} \cdot C \cdot V^m \cdot D^{m-1} \tag{3}$$

$$\bar{h} = N \cdot V^m \cdot D^{m-1} \qquad \text{where} \qquad N = \frac{k\, Pr^{1/3}}{\nu^m} C \tag{4,5}$$

Substituting into Eq. (2), the immersion error is

$$\Delta T_i(V, D, L) = (T_b - T_\infty)/\cosh\left\{\left[(4/k_g) N \cdot V^m \cdot D^{m-2}\right]^{1/2} L\right\} \tag{6} <$$

where k_g is the thermal conductivity of the glass thermometer.

(b) When the air velocity is 10 m/s, find

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.004\,m}{17.9 \times 10^{-6}\,m/s^2} = 2235$$

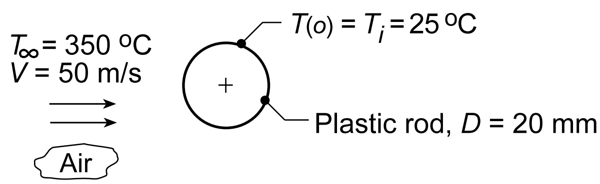
Continued...

PROBLEM 7.62

KNOWN: Long coated plastic, 20-mm diameter rod, initially at a uniform temperature of T_i = 25°C, is suddenly exposed to the cross-flow of air at T_∞ = 350°C and V = 50 m/s.

FIND: (a) Time for the surface of the rod to reach 175°C, the temperature above which the special coating cures, and (b) Compute and plot the time-to-reach 175°C as a function of air velocity for 5 ≤ V ≤ 50 m/s.

SCHEMATIC:



ASSUMPTIONS: (a) One-dimensional, transient conduction in the rod, (2) Constant properties, and (3) Evaluate thermophysical properties at $T_f = [(T_s + T_i)/2 + T_\infty] = [(175 + 25)/2 + 350]$°C = 225°C = 500 K.

PROPERTIES: Rod (Given): ρ = 2200 kg/m^3, c = 800 J/kg·K, k = 1 W/m·K, α = k/ρc = 5.68 × 10^{-7} m^2/s; *Table A.4*, Air (T_f ≈ 500 K, 1 atm): ν = 38.79 × 10^{-6} m^2/s, k = 0.0407 W/m·K, Pr = 0.684.

ANALYSIS: (a) To determine whether the lumped capacitance method is valid, determine the Biot number

$$Bi_{lc} = \frac{\bar{h}(r_o/2)}{k} \qquad (1)$$

The convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.63 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = 50\,m/s \times 0.020\,m \big/ 38.79\times10^{-6}\,m^2/s = 25,780$$

$$\bar{h} = \frac{0.0407\,W/m\cdot K}{0.020\,m}\left\{0.3 + \frac{0.63(25,780)^{1/2}(0.684)^{1/3}}{\left[1+(0.4/0.684)^{2/3}\right]^{1/4}}\left[1+\left(\frac{25,780}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 184\ W/m^2\cdot K \qquad (2)$$

Substituting for $\bar{h}$ from Eq. (2) into Eq. (1), find

$$Bi_{lc} = 184\,W/m^2\cdot K\,(0.010\,m/2)/1\,W/m\cdot K = 0.92 \qquad >> 0.1$$

Hence, the lumped capacitance method is inappropriate. Using the one-term series approximation, Section 5.6.2, Eqs. 5.49 with Table 5.1,

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) J_o\left(\zeta_1 r^*\right) \qquad r^* = r/r_o = 1$$

$$\theta^* = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = \frac{(175-350)^\circ C}{(25-350)^\circ C} = 0.54$$

$$Bi = \bar{h}r_o/k = 1.84 \qquad \zeta_1 = 1.5308\ rad \qquad C_1 = 1.3384$$

Continued...

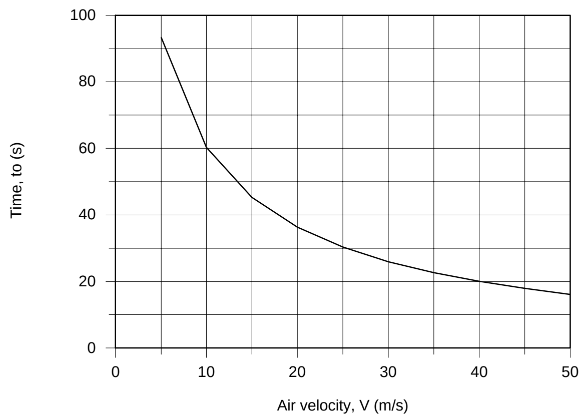
$$0.54 = 1.3384\exp[-(1.5308\text{rad})^2 Fo]J_o(1.5308 \times 1)$$

Using Table B.4 to evaluate $J_o(1.5308) = 0.4944$, find Fo = 0.0863 where

$$Fo = \frac{\alpha t_o}{r_o^2} = \frac{5.68\times10^{-7}\,m^2/s \times t_o}{(0.010\,m)^2} = 5.68\times10^{-3} t_o \qquad (6)$$

$$t_o = 15.2\,s \qquad <$$

(b) Using the *IHT Model*, *Transient Conduction*, *Cylinder*, and the *Tool*, *Correlations*, *External Flow*, *Cylinder*, results for the time-to-reach a surface temperature of 175°C as a function of air velocity V are plotted below.



COMMENTS: (1) Using the *IHT Tool*, *Correlations*, *External Flow*, *Cylinder*, the effect of the film temperature T_f on the estimated convection coefficient with V = 50 m/s can be readily evaluated.

T_f (K)	460	500	623
$\bar{h}$ ($W/m^2\cdot K$)	187	184	176

At early times, $\bar{h}$ = 184 $W/m^2\cdot K$ is a good estimate, while as the cylinder temperature approaches the airsteam temperature, the effect starts to be noticeable (10% decrease).

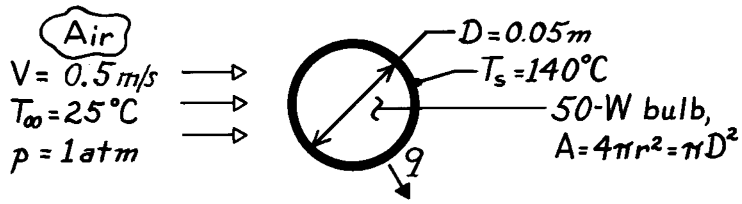
(2) The IHT analysis performed for part (b) was developed in two parts. Using a known value for $\bar{h}$, the *Transient Conduction*, *Cylinder Model* was tested. Separately, the *Correlation Tools* was assembled and tested. Then, the two files were merged to give the workspace for determining the time-to-reach 175°C as a function of velocity V.

PROBLEM 7.68

KNOWN: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

FIND: Heat loss by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (T_f = 25°C, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71, $\mu = 183.6 \times 10^{-7}$ N·s/m^2; *Table A-4*, Air (T_s = 140°C, 1 atm): $\mu = 235.5 \times 10^{-7}$ N·s/m^2.

ANALYSIS: The heat rate by convection is

$$q = \bar{h}\left(\pi D^2\right)\ \left(T_s - T_\infty\right)$$

where $\bar{h}$ may be estimated from the Whitaker relation

$$\bar{h} = \frac{k}{D}\left[2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}\right]$$

where

$$Re_D = \frac{VD}{\nu} = \frac{0.5\ \text{m/s} \times 0.05\ \text{m}}{15.71 \times 10^{-6}\ \text{m}^2/\text{s}} = 1591.$$

Hence,

$$\bar{h} = \frac{0.0261\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\ \left\{2 + \left[0.4(1591)^{1/2} + 0.06(1591)^{2/3}\right]\ (0.71)^{0.4}\left(\frac{183.6}{235.5}\right)^{1/4}\right\}$$

$$\bar{h} = 11.4\ \text{W/m}^2\cdot\text{K}$$

and the heat rate is

$$q = 11.4\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi\left(0.05\ \text{m}\right)^2\ \ (140 - 25)^\circ\text{C} = 10.3\ \text{W}.$$ <

COMMENTS: (1) The low value of $\bar{h}$ suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

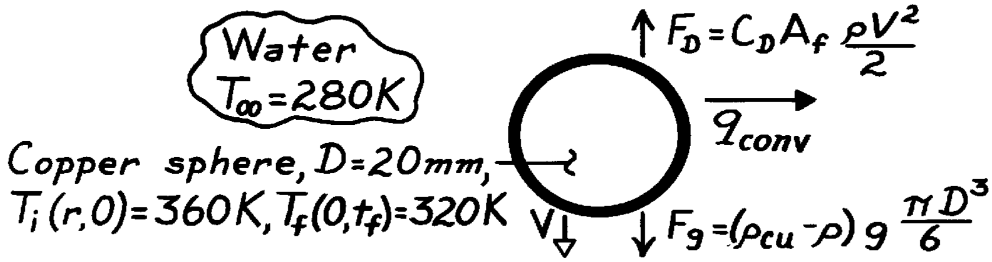
(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

PROBLEM 7.73

KNOWN: Diameter and initial and final temperatures of copper spheres quenched in a water bath.

FIND: (a) Terminal velocity in the bath, (b) Tank height.

SCHEMATIC:



ASSUMPTIONS: (1) Sphere descends at terminal velocity, (2) Uniform, but time varying surface, temperature.

PROPERTIES: *Table A-1*, Copper (350K): ρ = 8933 kg/m^3, k = 398 W/m·K, c_p = 387 J/kg·K; *Table A-6*, Water (T_∞ = 280 K): ρ = 1000 kg/m^3, μ = 1422 × 10^{-6} N·s/m^2, k = 0.582 W/m·K, Pr = 10.26; (T_s ≈ 340 K): μ_s = 420 × 10^{-5} N·s/m^2.

ANALYSIS: A force balance gives $C_D\left(p\,D^2/4\right)\,r V^2/2 = \left(r_{cu} - r\right)\, g\, p\, D^3/6,$

$$C_D\; V^2 = \frac{4D}{3}\frac{r_{cu}-r}{r}g = \frac{4\times 0.02\text{ m}}{3}\cdot\frac{8933-1000}{1000}9.8\text{ m/s}^2 = 2.07\text{ m}^2/\text{s}^2.$$

An iterative solution is needed, where C_D is obtained from Figure 7.8 with Re_D = VD/ν = 0.02 m V/1.42 × 10^{-6} m^2/s = 14,085 V (m/s). Convergence is achieved with

$$V \approx 2.1\text{ m/s}$$ <

for which Re_D = 29,580 and C_D ≈ 0.46. Using the Whitaker expression

$$\overline{Nu}_D = 2 + \left(0.4\times 29{,}850^{1/2} + 0.06\times 29{,}850^{2/3}\right)\,(10.26)^{0.4}(1422/420)^{1/4} = 439$$

$$\overline{h} = \overline{Nu}_D\;\; k/D = 439\times 0.582\text{ W/m}\cdot\text{K}/0.02\text{ m} = 12{,}775\text{ W/m}^2\cdot\text{K}.$$

To determine applicability of lumped capacitance method, find $Bi = \overline{h}\left(r_o/3\right)/\,k_{cu} = 12{,}775$ $\text{W/m}^2\cdot\text{K}\,\left(0.01\text{ m}/3\right)/398\text{W/m}\cdot\text{K} = 0.11$. Applicability is marginal. Use Heisler charts,

$$q_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{320-280}{360-280} = 0.5, \quad Bi^{-1} = \frac{k}{\overline{h}r_o} = 3.12, \quad Fo \approx 0.88 = \frac{a\,t_f}{r_o^2}.$$

With α_{cu} = k/ρc_p = 398 W/m·K/(8933 kg/m^3) (387 J/kg·K) = 1.15 × 10^{-4} m^2/s, find

$$t_f = 0.88\;(0.01\text{ m})^2/1.15\times 10^{-4}\text{m}^2/\text{s} = 0.77\text{ s}.$$

Required tank height is

$$H = t_f\cdot V = 0.77\text{ s}\times 2.1\text{ m/s} = 1.6\text{ m}.$$ <

COMMENTS: If t_f is evaluated from the approximate series solution, $q_o^* = C_1\exp\left(-z_1^2\;Fo\right)$, we obtain t_f = 0.76 s. Note that the terminal velocity is not reached immediately. Reduced V implies reduced $\overline{h}$ and increased t_f.