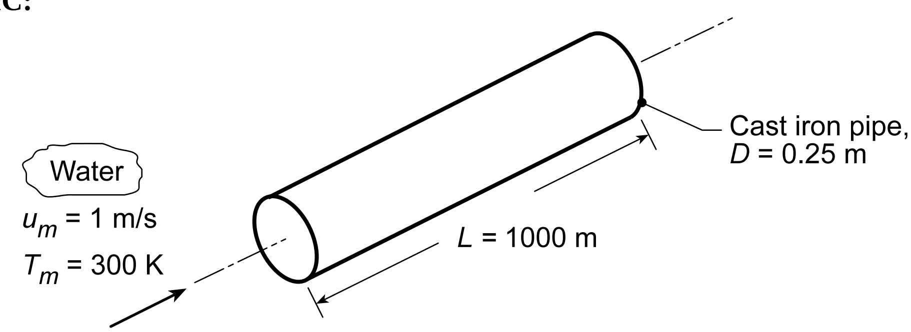
## PROBLEM 8.3

**KNOWN:** Temperature and velocity of water flow in a pipe of prescribed dimensions.

**FIND:** Pressure drop and pump power requirement for (a) a smooth pipe, (b) a cast iron pipe with a clean surface, and (c) smooth pipe for a range of mean velocities 0.05 to 1.5 m/s.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady, fully developed flow.

**PROPERTIES:** *Table A.6*, Water (300 K): $\rho = 997\ \text{kg/m}^3$, $\mu = 855 \times 10^{-6}\ \text{N·s/m}^2$, $\nu = \mu/\rho = 8.576 \times 10^{-7}\ \text{m}^2\text{/s}$.

**ANALYSIS:** From Eq. 8.22a and 8.22b, the pressure drop and pump power requirement are

$$\Delta p = f\,\frac{\rho u_m^2}{2D}\,L \qquad\qquad P = \Delta p \dot{V} = \Delta p\left(\pi D^2/4\right)u_m \tag{1,2}$$

The friction factor, f, may be determined from Figure 8.3 for different relative roughness, e/D, surfaces or from Eq. 8.21 for the smooth condition, $3000 \le Re_D \le 5 \times 10^6$,

$$f = \left(0.790\ln\left(Re_D\right) - 1.64\right)^{-2} \tag{3}$$

where the Reynolds number is

$$Re_D = \frac{u_m D}{\nu} = \frac{1\,\text{m/s} \times 0.25\,\text{m}}{8.576 \times 10^{-7}\ \text{m}^2/\text{s}} = 2.915 \times 10^5 \tag{4}$$

(a) *Smooth surface:* from Eqs. (3), (1) and (2),

$$f = \left(0.790\ln\left(2.915 \times 10^5\right) - 1.64\right)^{-2} = 0.01451$$

$$\Delta p = 0.01451\left(997\,\text{kg/m}^3 \times 1\,\text{m}^2/\text{s}^2 / 2 \times 0.25\,\text{m}\right)1000\,\text{m} = 2.89 \times 10^4\ \text{kg/s}^2\cdot\text{m} = 0.289\,\text{bar} \quad <$$

$$P = 2.89 \times 10^4\ \text{N/m}^2\left(\pi \times 0.25^2\ \text{m}^2/4\right)1\,\text{m/s} = 1418\,\text{N}\cdot\text{m/s} = 1.42\,\text{kW} \quad <$$

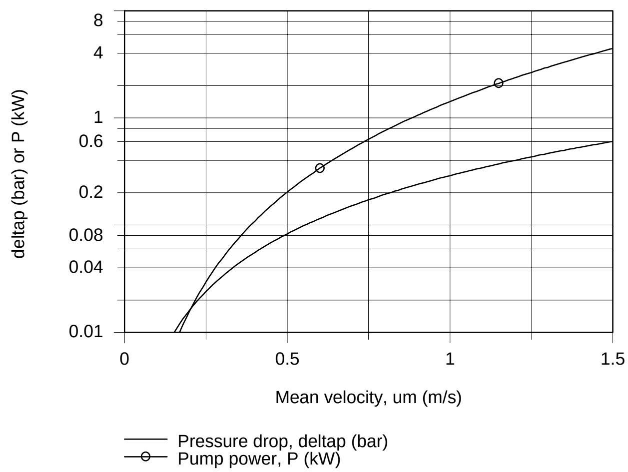
(b) *Cast iron clean surface:* with e = 260 μm, the relative roughness is $e/D = 260 \times 10^{-6}\ \text{m}/0.25\ \text{m} = 1.04 \times 10^{-3}$. From Figure 8.3 with $Re_D = 2.92 \times 10^5$, find f = 0.021. Hence,

$\Delta p = 0.419$ bar $\qquad\qquad P = 2.06$ kW $\quad <$

(c) *Smooth surface:* Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of mean velocity, $u_m$, for the range $0.05 \le u_m \le 1.5$ m/s are computed and plotted below.

Continued...

# PROBLEM 8.3 (Cont.)




The pressure drop is a strong function of the mean velocity. So is the pump power since it is proportional to both $\Delta p$ and the mean velocity.

**COMMENTS:** (1) Note that L/D = 4000 >> $(x_{fg,h}/D) \approx 10$ for turbulent flow and the assumption of fully developed conditions is justified.

(2) Surface fouling results in increased surface roughness and increases operating costs through increasing pump power requirements.

(3) The *IHT Workspace* used to generate the graphical results follows.

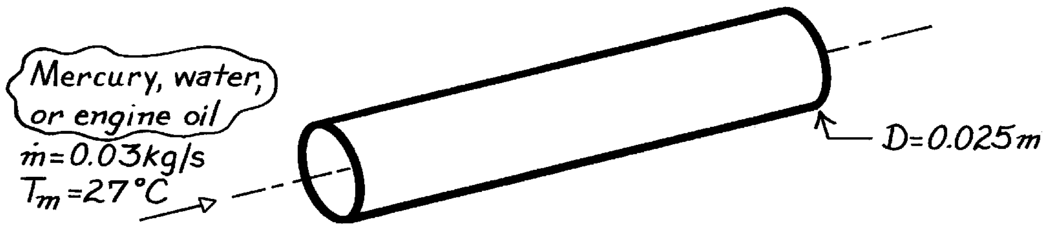
```
// Pressure drop:
deltap = f * rho * um^2 * L / ( 2 * D )          // Eq (1); Eq 8.22a
deltap_bar = deltap / 1.00e5                     // Conversion, Pa to bar units
Power = deltap * ( pi * D^2 / 4 ) * um           // Eq (2); Eq 8.22b
Power_kW = Power / 1000                          // Useful for scaling graphical result

// Reynolds number and friction factor:
ReD = um * D / nu                                // Eq (3)
f =  (0.790 * ln (ReD) - 1.64 ) ^ (-2)           // Eq (4); Eq 8.21, smooth surface condition

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                          // Quality (0=sat liquid or 1=sat vapor)
rho = rho_Tx("Water",Tm,x)     // Density, kg/m^3
nu = nu_Tx("Water",Tm,x)       // Kinematic viscosity, m^2/s

// Assigned variables:
um = 1                         // Mean velocity, m/s
Tm = 300                       // Mean temperature, K
D = 0.25                       // Tube diameter, m
L = 1000                       // Tube length, m
```

# PROBLEM 8.4

**KNOWN:** Temperature and mass flow rate of various liquids moving through a tube of prescribed diameter.

**FIND:** Mean velocity and hydrodynamic and thermal entry lengths.

**SCHEMATIC:**




**ASSUMPTIONS:** Constant properties.

**PROPERTIES:** (T = 300K)

| *Liquid* | *Table* | *r(kg/m3)* | *m(N·s/m2)* | *n(m2/s)* | *Pr* |
|---|---|---|---|---|---|
| Engine oil | A-5 | 884 | 0.486 | $550 \times 10^{-6}$ | 6400 |
| Mercury | A-5 | 13,529 | $0.152 \times 10^{-2}$ | $0.113 \times 10^{-6}$ | 0.0248 |
| Water | A-6 | 1000 | $0.855 \times 10^{-3}$ | $0.855 \times 10^{-6}$ | 5.83 |

**ANALYSIS:** The mean velocity is given by

$$u_m = \frac{\dot{m}}{r A_c} = \frac{0.03\ \text{kg/s}}{r p\,(0.025\text{m})^2 / 4} = \frac{61.1\ \text{kg/s}\cdot\text{m}^2}{r}.$$

The hydrodynamic and thermal entry lengths depend on $Re_D$,

$$Re_D = \frac{4\dot{m}}{p\ D m} = \frac{4 \times 0.03\ \text{kg/s}}{p\,(0.025\text{m})\, m} = \frac{1.53\ \text{kg/s}\cdot\text{m}}{m}.$$

Hence, even for water ($\mu = 0.855 \times 10^{-3}$ N·s/m$^2$), $Re_D < 2300$ and the flow is laminar. From Eqs. 8.3 and 8.23 it follows that

$$x_{fd,h} = 0.05\ D\ Re_D = \frac{1.91 \times 10^{-3}\ \text{kg/s}}{m}$$

$$x_{fd,t} = 0.05\ D\ Re_D\ Pr = \frac{\left(1.91 \times 10^{-3}\ \text{kg/s}\right) Pr}{m}.$$

Hence:

| *Liquid* | $u_m$*(m/s)* | $x_{fd,h}$*(m)* | $x_{fd,t}$*(m)* |
|---|---|---|---|
| Oil | 0.069 | 0.0039 | 25.2 |
| Mercury | 0.0045 | 1.257 | 0.031 |
| Water | 0.061 | 2.234 | 13.02 |

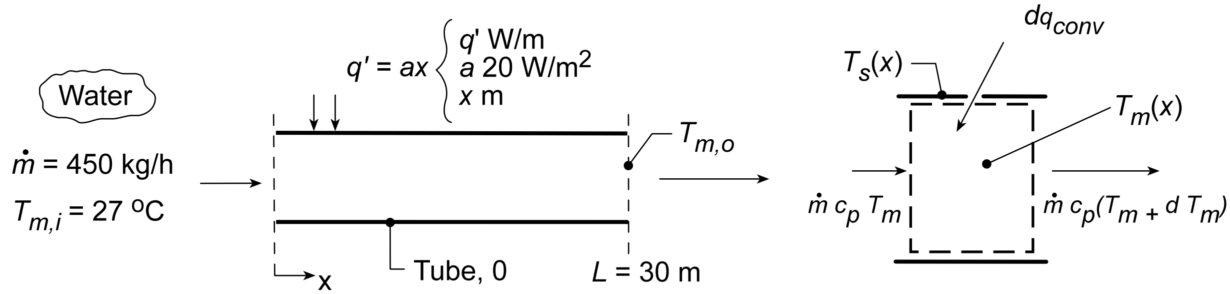
**COMMENTS:** Note the effect of viscosity on the hydrodynamic entry length and the effect of Pr on the thermal entry length.

## PROBLEM 8.11

**KNOWN:** Internal flow with prescribed wall heat flux as a function of distance.

**FIND:** (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed *and* developing flow conditions, and (d) Value of uniform wall flux $q_s''$ (instead of $q_s' = ax$) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Incompressible flow.

**PROPERTIES:** *Table A.6*, Water (300 K): $c_p$ = 4.179 kJ/kg·K.

**ANALYSIS:** (a) Applying energy conservation to the control volume above,

$$dq_{conv} = \dot{m}c_p dT_m \qquad (1)$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{conv} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}. \qquad (2)$$

Separating and integrating with proper limits gives

$$a\int_{x=0}^{x} x dx = \dot{m}c_p \int_{T_{m,i}}^{T_m(x)} dT_m \qquad T_m(x) = T_{m,i} + \frac{ax^2}{2\dot{m}c_p} \qquad (3,4) <$$

(b) To find the outlet temperature, let x = L, then

$$T_m(L) = T_{m,o} = T_{m,i} + aL^2 \big/ 2\dot{m}c_p. \qquad (5)$$

Solving for $T_{m,o}$ and substituting numerical values, find

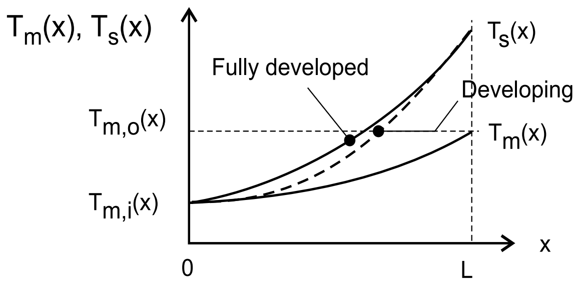
$$T_{m,o} = 27^\circ C + \frac{20\, W/m^2 \left(30\, m^2\right)}{2\left(450\, kg/h / (3600\, s/h)\right) \times 4179\, J/kg \cdot K} = 27^\circ C + 17.2^\circ C = 44.2^\circ C. \quad <$$

(c) For *linear wall heating*, $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x) \cdot \pi D \left(T_s(x) - T_m(x)\right) \qquad (6)$$

For fully developed flow conditions, h(x) = h is a constant; hence, $T_s(x) - T_m(x)$ increases linearly with x. For developing conditions, h(x) will decrease with increasing distance along the tube eventually achieving the fully developed value.

Continued...



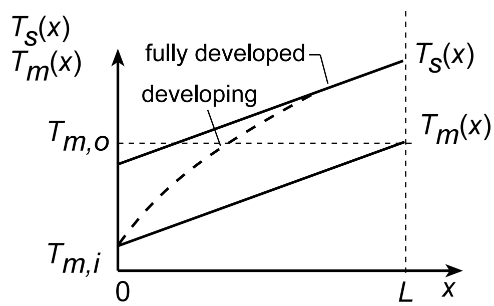

(d) For *uniform wall heat flux heating*, the overall energy balance on the tube yields

$$q = q_s''\pi DL = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

Requiring that $T_{m,o}$ = 44.2°C from part (a), find

$$q_s'' = \frac{(450/3600)\,\text{kg/s}\times 4179\,\text{J/kg}\cdot\text{K}\,(44.2-27)\,\text{K}}{\pi D\times 30\,\text{m}} = 95.3/D\ \text{W/m}^2 \qquad <$$

where D is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, $q_s''$. For uniform heating, Section 3.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.




**COMMENTS:** (1) Note that $c_p$ should be evaluated at $T_m$ = (27 + 44)°C/2 = 309 K.

(2) Why did we show $T_s(0) = T_m(0)$ for both types of history when the flow was developing?

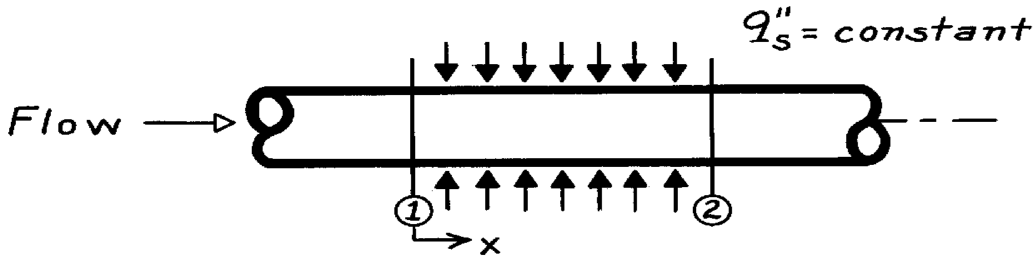
(3) Why must $T_m(x)$ be linear with distance in the case of uniform wall flux heating?

**PROBLEM 8.12**

**KNOWN:** Internal flow with constant surface heat flux, $q_s''$.

**FIND:** (a) Qualitative temperature distributions, T(x), under developing and fully-developed flow, (b) Exit mean temperature for both situations.

**SCHEMATIC:**




**ASSUMPTIONS:** (a) Steady-state conditions, (b) Constant properties, (c) Incompressible flow.

**ANALYSIS:** Based upon the analysis leading to Eq. 8.40, note for the case of constant surface heat flux conditions,

$$\frac{dT_m}{dx} = \text{constant.}$$

Hence, regardless of whether the hydrodynamic or thermal boundary layer is fully developed, it follows that

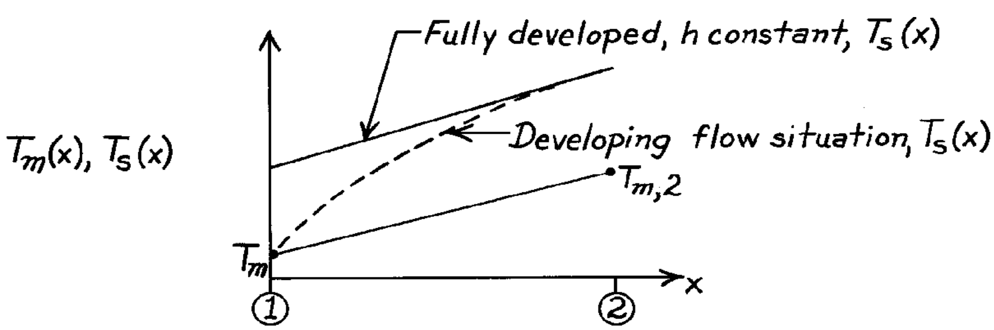
$T_m(x)$ is linear and

$T_{m,2}$ will be the same for all flow conditions. <

The surface heat flux can also be written, using Eq. 8.28, as

$$q_s'' = h\left[T_s(x) - T_m(x)\right].$$

Under fully-developed flow and thermal conditions, $h = h_{fd}$ is a constant. When flow is developing $h > h_{fd}$. Hence, the temperature distributions appear as below.



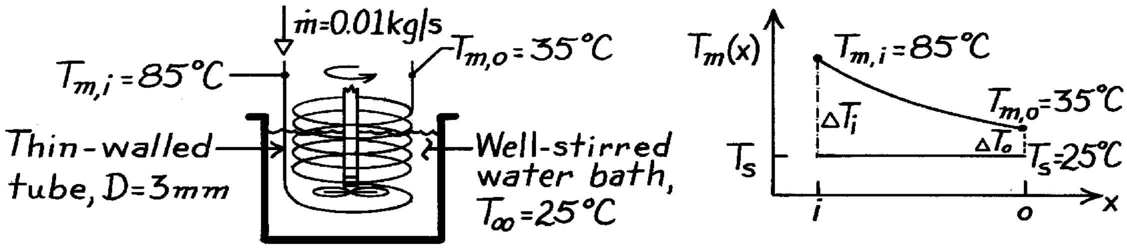

<

## PROBLEM 8.26

**KNOWN:** Ethylene glycol flowing through a coiled, thin walled tube submerged in a well-stirred water bath maintained at a constant temperature.

**FIND:** Heat rate and required tube length for prescribed conditions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Convection coefficient on water side infinite; cooling process approximates constant wall surface temperature distribution, (4) KE, PE and flow work changes negligible, (5) Constant properties, (6) Negligible heat transfer enhancement associated with the coiling.

**PROPERTIES:** *Table A-5*, Ethylene glycol ($T_m$ = (85 + 35)°C/2 = 60°C = 333 K): $c_p$ = 2562 J/kg·K, $\mu = 0.522 \times 10^{-2}$ N·s/m$^2$, k = 0.260 W/m·K, Pr = 51.3.

**ANALYSIS:** From an overall energy balance on the tube,

$$q_{conv} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.01\ \text{kg/s} \times 2562\ \text{J/kg}\left(35-85\right)^\circ \text{C} = -1281\ \text{W}. \qquad (1) <$$

For the constant surface temperature condition, from the rate equation,

$$A_s = q_{conv} / \bar{h}\Delta T_{\ell m} \qquad (2)$$

$$\Delta T_{\ell m} = \left(\Delta T_o - \Delta T_i\right) / \ell n \frac{\Delta T_o}{\Delta T_i} = \left[\left(35-25\right)^\circ \text{C} - \left(85-25\right)^\circ \text{C}\right] / \ell n \frac{35-25}{85-25} = 27.9^\circ \text{C}. \qquad (3)$$

Find the Reynolds number to determine flow conditions,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01\ \text{kg/s}}{\pi \times 0.003\ \text{m} \times 0.522 \times 10^{-2}\ \text{N}\cdot\text{s/m}^2} = 813. \qquad (4)$$

Hence, the flow is laminar and, assuming the flow is fully developed, the appropriate correlation is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 3.66, \qquad \bar{h} = Nu\frac{k}{D} = 3.66 \times 0.260 \frac{\text{W}}{\text{m}\cdot\text{K}} / 0.003\text{m} = 317\ \text{W/m}^2\cdot\text{K}. \qquad (5)$$

From Eq. (2), the required area, $A_s$, and tube length, L, are

$$A_s = 1281\ \text{W} / 317\ \text{W/m}^2\cdot\text{K} \times 27.9^\circ\text{C} = 0.1448\ \text{m}^2$$

$$L = A_s / \pi D = 0.1448\text{m}^2 / \pi\left(0.003\text{m}\right) = 15.4\text{m}. \qquad <$$

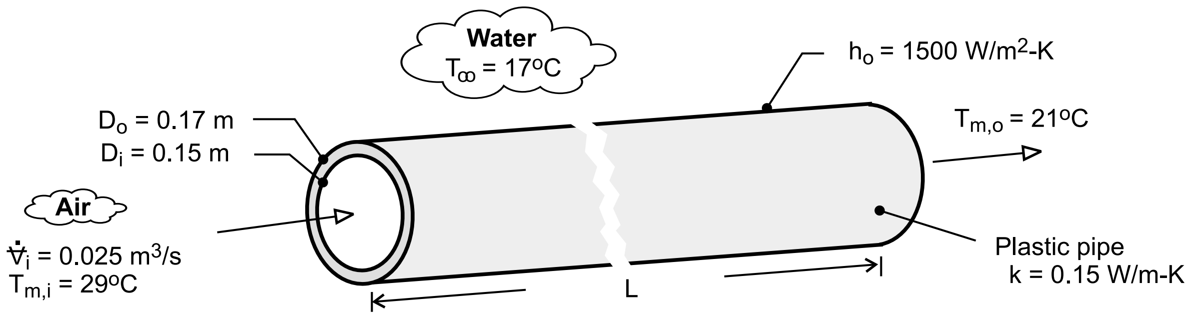
**COMMENTS:** Note that for fully developed laminar flow conditions, the requirement is satisfied: $Gz^{-1}$ = (L/D) / $Re_D$ Pr = (15.3/0.003) / (813 × 51.3) = 0.122 > 0.05. Note also the sign of the heat rate $q_{conv}$ when using Eqs. (1) and (2).

# PROBLEM 8.32

**KNOWN:** Thermal conductivity and inner and outer diameters of plastic pipe. Volumetric flow rate and inlet and outlet temperatures of air flow through pipe. Convection coefficient and temperature of water.

**FIND:** Pipe length and fan power requirement.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Negligible heat transfer from air in vertical legs of pipe, (3) Negligible flow work and potential and kinetic energy changes for air flow through pipe, (4) Smooth interior surface, (5) Constant properties.

**PROPERTIES:** *Table A-4*, Air ($T_{m,i}$ = 29°C): $\rho_i = 1.155\,\text{kg/m}^3$. Air $(\overline{T}_m = 25°\text{C})$: $c_p$ = 1007 J/kg·K, $\mu = 183.6 \times 10^{-7}$ N·s/m$^2$, $k_a$ = 0.0261 W/m·K, Pr = 0.707.

**ANALYSIS:** From Eq. (8.46a)

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}\,c_p}\right)$$

where, from Eq. (3.32), $\left(\overline{U}A_s\right)^{-1} = R_{tot} = \dfrac{1}{\overline{h}_i \pi D_i L} + \dfrac{\ln\left(D_o/D_i\right)}{2\pi L k} + \dfrac{1}{h_o \pi D_o L}$

With $\dot{m} = \rho_i \dot{\forall}_i = 0.0289\,\text{kg/s}$ and $Re_D = 4\dot{m}/\pi D_i \mu = 13,350$, flow in the pipe is turbulent. Assuming fully developed flow throughout the pipe, and from Eq. (8.60),

$$\overline{h}_i = \frac{k_a}{D_i} 0.023 Re_D^{4/5} Pr^{0.3} = \frac{0.0261\,\text{W/m}\cdot\text{K} \times 0.023}{0.15\text{m}} (13,350)^{4/5} (0.707)^{0.3} = 7.20\,\text{W/m}^2\cdot\text{K}$$

$$\left(\overline{U}A_s\right)^{-1} = \frac{1}{L}\left(\frac{1}{7.21\,\text{W/m}^2\cdot\text{K} \times \pi \times 0.15\text{m}} + \frac{\ln(0.17/0.15)}{2\pi \times 0.15\,\text{W/m}\cdot\text{K}} + \frac{1}{1500\,\text{W/m}^2\cdot\text{K} \times \pi \times 0.17\text{m}}\right)$$

$$\overline{U}A_s = \frac{L}{(0.294 + 0.133 + 0.001)} = 2.335\,L\ \text{W/K}$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \frac{17-21}{17-29} = 0.333 = \exp\left(-\frac{2.335\,L}{0.0289\,\text{kg/s} \times 1007\,\text{J/kg}\cdot\text{K}}\right) = \exp(-0.0802)$$

$$L = -\frac{\ln(0.333)}{0.0802} = 13.7\text{m}$$ <

From Eqs. (8.22a) and (8.22b) and with $u_{m,i} = \dot{\forall}_i / \left(\pi D_i^2/4\right) = 1.415\,\text{m/s}$, the fan power is

$$P = (\Delta p)\dot{\forall} \approx f\,\frac{\rho_i u_{m,i}^2}{2D_i} L\,\dot{\forall}_i = 0.0294\,\frac{1.155\,\text{kg/m}^3\,(1.415\,\text{m/s})^2}{2(0.15\text{m})} 13.7\text{m} \times 0.025\,\text{m}^3/\text{s} = 0.078\,\text{W}$$ <

where $f = 0.316\,Re_D^{-1/4} = 0.0294$ from Eq. (8.20a).

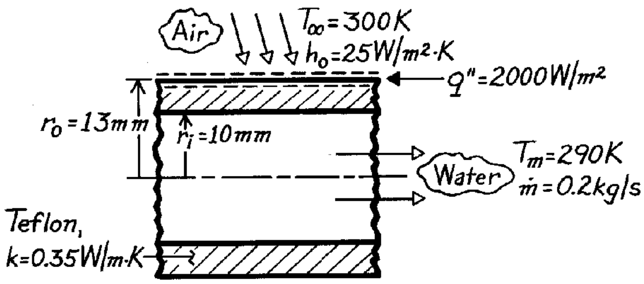
**COMMENTS:** (1) With L/Di = 91, the assumption of fully developed flow throughout the pipe is justified. (2) The fan power requirement is small, and the process is economical. (3) The resistance to heat transfer associated with convection at the outer surface is negligible.

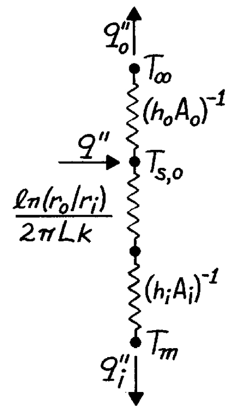
## PROBLEM 8.70

**KNOWN:** Inner and outer radii and thermal conductivity of a teflon tube. Flowrate and temperature of confined water. Heat flux at outer surface and temperature and convection coefficient of ambient air.

**FIND:** Fraction of heat transfer to water and temperature of tube outer surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Fully-developed flow, (3) One-dimensional conduction, (4) Negligible tape contact and conduction resistances.

**PROPERTIES:** *Table A-6*, Water ($T_m$ = 290K): $\mu = 1080 \times 10^{-6}$ kg/s·m, k = 0.598 W/m·K, Pr = 7.56.

**ANALYSIS:** The outer surface temperature follows from a surface energy balance

$$(2\pi r_o L)q'' = \frac{T_{s,o} - T_\infty}{(h_o 2\pi r_o L)^{-1}} + \frac{T_{s,o} - T_m}{(\ln(r_o/r_i)/2\pi Lk) + (1/2\pi r_i L h_i)}$$

$$q'' = h_o(T_{s,o} - T_\infty) + \frac{T_{s,o} - T_m}{(r_o/k)\ln(r_o/r_i) + (r_o/r_i)/h_i}.$$

With $\quad Re_D = 4\dot{m}/(\pi D\mu) = 4(0.2\text{kg/s})/\left[\pi(0.02\text{ m})1080 \times 10^{-6}\text{ kg/s}\cdot\text{m}\right] = 11{,}789$

the flow is turbulent and Eq. 8.60 yields

$$h_i = (k/D_i)0.023Re_D^{4/5}Pr^{0.4} = (0.598\text{ W/m}\cdot\text{K}/0.02\text{ m})(0.023)(11{,}789)^{4/5}(7.56)^{0.4} = 2792\text{ W/m}^2\cdot\text{K}.$$

Hence

$$2000\text{ W/m}^2 = 25\text{ W/m}^2\cdot\text{K}(T_{s,o} - 300\text{K}) + \frac{T_{s,o} - 290\text{ K}}{(0.013\text{ m}/0.35\text{ W/m}\cdot\text{K})\ln(1.3) + (1.3)/(2792\text{ W/m}^2\cdot\text{K})}$$

and solving for $T_{s,o}$, $\qquad T_{s,o} = 308.3$ K. $\qquad$ <

The heat flux to the air is

$$q''_o = h_o(T_{s,o} - T_\infty) = 25\text{ W/m}^2\cdot\text{K}(308.3 - 300)\text{K} = 207.5\text{ W/m}^2.$$

Hence, $\qquad q''_i/q'' = (2000 - 207.5)\text{ W/m}^2/2000\text{ W/m}^2 = 0.90.$ $\qquad$ <

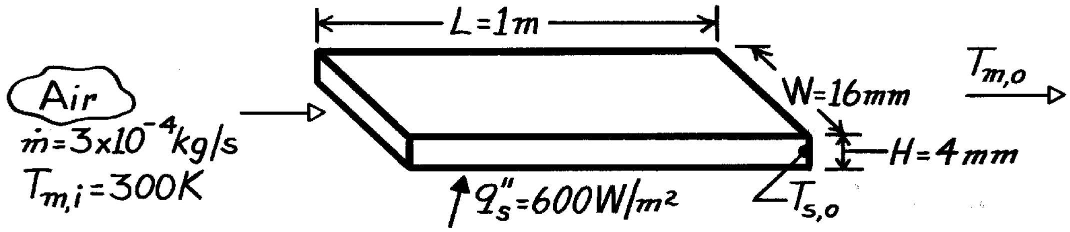
**COMMENTS:** The resistance to heat transfer by convection to the air substantially exceeds that due to conduction in the teflon and convection in the water. Hence, most of the heat is transferred to the water.

# PROBLEM 8.77

**KNOWN:** Flow rate and inlet temperature of air passing through a rectangular duct of prescribed dimensions and surface heat flux.

**FIND:** Air and duct surface temperatures at outlet.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform surface heat flux, (3) Constant properties, (4) Atmospheric pressure, (3) Fully developed conditions at duct exit, (6) Negligible KE, PE and flow work effects.

**PROPERTIES:** *Table A-4*, Air $\left(\bar{T}_m \approx 300\text{K, 1 atm}\right)$: $c_p = 1007$ J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m$^2$, k = 0.0263 W/m·K, Pr = 0.707.

**ANALYSIS:** For this uniform heat flux condition, the heat rate is

$$q = q_s'' A_s = q_s''\left[2(L\times W) + 2(L\times H)\right]$$

$$q = 600\ \text{W/m}^2\left[2(1\text{m}\times 0.016\text{m}) + 2(1\text{m}\times 0.004\text{m})\right] = 24\ \text{W}.$$

From an overall energy balance

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}\, c_p} = 300\text{K} + \frac{24\ \text{W}}{3\times 10^{-4}\ \text{kg/s}\times 1007\ \text{J/kg}\cdot\text{K}} = 379\ \text{K}. \qquad <$$

The surface temperature at the outlet may be determined from Newton's law of cooling, where

$$T_{s,o} = T_{m,o} + q''/h.$$

From Eqs. 8.67 and 8.1

$$D_h = \frac{4\, A_c}{P} = \frac{4(0.016\text{m}\times 0.004\text{m})}{2(0.016\text{m} + 0.004\text{m})} = 0.0064\ \text{m}$$

$$Re_D = \frac{r\ u_m\ D_h}{m} = \frac{\dot{m}\, D_h}{A_c m} = \frac{3\times 10^{-4}\text{kg/s}\,(0.0064\text{m})}{64\times 10^{-6}\text{m}^2\left(184.6\times 10^{-7}\ \text{N}\cdot\text{s/m}^2\right)} = 1625.$$

Hence the flow is laminar, and from Table 8.1

$$h = \frac{k}{D_h}5.33 = \frac{0.0263\ \text{W/m}\cdot\text{K}}{0.0064\ \text{m}}5.33 = 22\ \text{W/m}^2\cdot\text{K}$$

$$T_{s,o} = 379\ \text{K} + \frac{600\ \text{W/m}^2}{22\ \text{W/m}^2\cdot\text{K}} = 406\ \text{K}. \qquad <$$

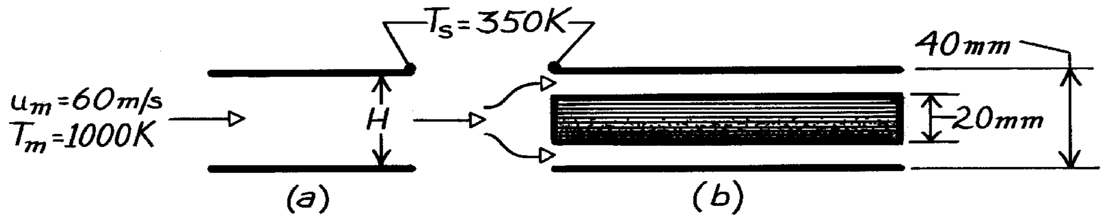
**COMMENTS:** The calculations should be reperformed with properties evaluated at $\bar{T}_m = 340$ K. The change in $T_{m,o}$ would be negligible, and $T_{s,o}$ would decrease slightly.

# PROBLEM 8.79

**KNOWN:** Temperature and velocity of gas flow between parallel plates of prescribed surface temperature and separation. Thickness and location of plate insert.

**FIND:** Heat flux to the plates (a) without and (b) with the insert.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible radiation, (3) Gas has properties of atmospheric air, (4) Plates are of infinite width W, (5) Fully developed flow.

**PROPERTIES:** *Table A-4*, Air (1 atm, $T_m$ = 1000K): $\rho = 0.348$ kg/m$^3$, $\mu = 424.4 \times 10^{-7}$ kg/s·m, k = 0.0667 W/m·K, Pr = 0.726.

**ANALYSIS:** (a) Based upon the hydraulic diameter $D_h$, the Reynolds number is

$$D_h = 4\,A_c / P = 4(H \cdot W)/2(H + W) = 2H = 80 \text{ mm}$$

$$Re_{D_h} = \frac{\rho\, u_m\, D_h}{\mu} = \frac{0.348 \text{ kg/m}^3 (60 \text{ m/s}) 0.08 \text{ m}}{424.4 \times 10^{-7} \text{ kg/s} \cdot \text{m}} = 39{,}360.$$

Since the flow is fully developed and turbulent, use the Dittus-Boelter correlation,

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{0.3} = 0.023(39{,}360)^{4/5}(0.726)^{0.3} = 99.1$$

$$h = \frac{k}{D_h} Nu_D = \frac{0.0667 \text{ W/m} \cdot \text{K}}{0.08 \text{ m}} 99.1 = 82.6 \text{ W/m}^2 \cdot \text{K}$$

$$q'' = h(T_m - T_s) = 82.6 \text{ W/m}^2 \cdot \text{K}(1000 - 350)\text{K} = 53{,}700 \text{ W/m}^2. \qquad <$$

(b) From continuity,

$$\dot{m} = (\rho\, u_m A)_a = (\rho\, u_m A)_b \qquad u_m)_b = u_m)_a (\rho A)_a / (\rho A)_b = 60 \text{ m/s}(40/20) = 120 \text{ m/s}.$$

For each of the resulting channels, $D_h$ = 0.02 m and

$$Re_{D_h} = \frac{\rho\, u_m\, D_h}{\mu} = \frac{0.348 \text{ kg/m}^3 (120 \text{ m/s})\, 0.02 \text{ m}}{424.4 \times 10^{-7} \text{kg/s} \cdot \text{m}} = 19{,}680.$$

Since the flow is still turbulent,

$$Nu_D = 0.023(19{,}680)^{4/5}(0.726)^{0.3} = 56.9 \qquad h = \frac{56.9(0.0667 \text{ W/m} \cdot \text{K})}{0.02 \text{ m}} = 189.8 \text{ W/m}^2 \cdot \text{K}$$

$$q'' = 189.8 \text{ W/m}^2 \cdot \text{K}\ (1000 - 350)\text{K} = 123{,}400 \text{ W/m}^2. \qquad <$$

**COMMENTS:** From the Dittus-Boelter equation,

$$h_b / h_a = (u_{m,b} / u_{m,a})^{0.8} (D_{h,a} / D_{h,b})^{0.2} = (2)^{0.8}(4)^{0.2} = 1.74 \times 1.32 = 2.30.$$

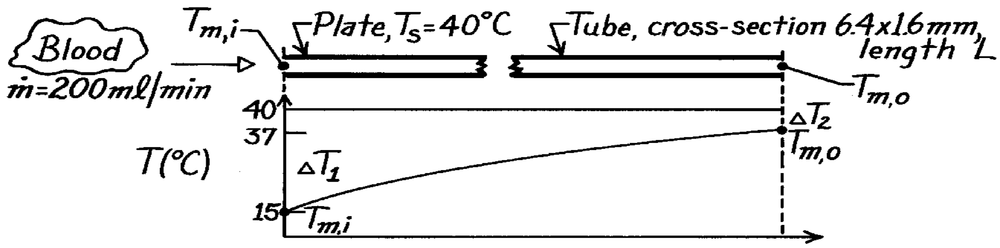
Hence, heat transfer enhancement due to the insert is primarily a result of the increase in $u_m$ and secondarily a result of the decrease in $D_h$.

# PROBLEM 8.82

**KNOWN:** Heat exchanger to warm blood from a storage temperature 10°C to 37° at 200 ml/min. Tubing has rectangular cross-section 6.4 mm × 1.6 mm sandwiched between plates maintained at 40°C.

**FIND:** (a) Length of tubing and (b) Assessment of assumptions to indicate whether analysis under- or over-estimates length.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Blood flow is fully developed, (4) Blood has properties of water, and (5) Negligible tube wall and contact resistance.

**PROPERTIES:** *Table A-6*, Water ($\overline{T}_m \approx 300$ K): $c_{p,f}$ = 4179 J/kg·K, $\rho_f = 1/v_f$ = 997 kg/m$^3$, $\nu_f$ = $\mu_f v_f = 8.58 \times 10^{-7}$ m$^2$/s, k = 0.613 W/m·K, Pr = 5.83.

**ANALYSIS:** (a) From an overall energy balance and the rate equation,

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = \overline{h} A_s \Delta T_{LMTD} \qquad (1)$$

where

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} = \frac{(40-15)-(40-37)}{\ln(25/3)} = 10^\circ\text{C}.$$

To estimate $\overline{h}$, find the Reynolds number for the rectangular tube,

$$Re_D = \frac{u_m D_h}{n} = \frac{0.326 \text{ m/s} \times 0.00256 \text{ m}}{8.58 \times 10^{-7} \text{m}^2/\text{s}} = 973$$

where

$$D_h = 4\,A_c/P = 4(6.4 \text{ mm} \times 1.6 \text{ mm})/2(6.4+1.6)\text{mm} = 2.56 \text{ mm}$$

$$A_c = (6.4 \text{ mm} \times 1.6 \text{ mm}) = 1.024 \times 10^{-5} \text{m}^2$$

$$u_m = \dot{m}/r A_c = \dot{\forall}/A_c = 200 \text{ m}\ell/60 \text{ s}\left(10^{-6}\text{m}^3/\text{m}\ell\right)/1.024\times10^{-5}\text{m}^2 = 0.326 \text{ m/s}.$$

Hence the flow is laminar, but assuming fully developed flow with an isothermal surface from Table 8.1 with b/a = 6.4/1.6 = 4,

$$Nu_D = \frac{hD_h}{k} = 4.4 \qquad h = \frac{4.4 \times 0.613 \text{ W/m}\cdot\text{K}}{0.00256 \text{ m}} = 1054 \text{ W/m}^2\cdot\text{K}.$$

Continued …..

From Eq. (1) with

$$A_s = PL = 2(6.4 + 1.6)\times 10^{-3} m \times L = 1.6\times 10^{-2} Lz$$

$$\dot{m} = \rho A_c u_m = 997 \text{ kg/m}^3 \times 1.024\times 10^{-5} \text{ m}^2 \times 0.326 \text{ m/s} = 3.328\times 10^{-3} \text{ kg/s}$$

the length of the rectangular tubing can be found as

$$3.328\times 10^{-3} \text{ kg/s} \times 4179 \text{ J/kg}\cdot\text{K}(37-15)\text{K} = 1054 \text{ W/m}^2\cdot\text{K}\times 1.6\times 10^{-2} L\text{m}^2 \times 10 \text{ K}$$

$$L = 1.8 \text{ m.}$$ <

(b) Consider these comments with regard to whether the analysis under- or over-estimates the length,

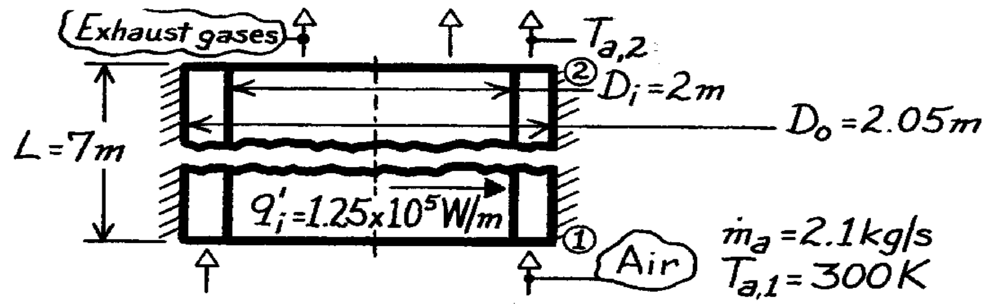
- ⇒ fully-developed flow - $L/D_h$ = 1.8 m/0.00256 = 700; not likely to have any effect,
- ⇒ negligible tube wall resistance - depends upon materials of construction; if plastic, analysis under predicts length,
- ⇒ negligible thermal contact resistance between tube and heating plate - if present, analysis under predicts length.

## PROBLEM 8.97

**KNOWN:** Heat rate per unit length at the inner surface of an annular recuperator of prescribed dimensions. Flow rate and inlet temperature of air passing through annular region.

**FIND:** (a) Temperature of air leaving the recuperator, (b) Inner pipe temperature at inlet and outlet and outer pipe temperature at inlet.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Uniform heating of recuperator inner surface, (4) Adiabatic outer surface, (5) Negligible kinetic and potential energy changes for air, (6) Fully developed air flow throughout.

**PROPERTIES:** *Table A-4*, Air $\left(\overline{T}_m = 500K\right)$: $c_p$ = 1030 J/kg·K, μ = 270 × $10^{-7}$ N·s/m$^2$, k = 0.041 W/m·K, Pr = 0.68.

**ANALYSIS:** (a) From an energy balance on the air

$$q_i' L = \dot{m}_a \; c_{p,a} \left( T_{a,2} - T_{a,1} \right)$$

$$T_{a,2} = T_{a,1} + \frac{q_i' \; L}{\dot{m}_a \; c_{p,a}} = 300K + \frac{1.25 \times 10^5 W/m \times 7m}{2.1 \text{ kg/s} \times 1030 \text{ J/kg} \cdot K} = 704.5K.$$ <

(b) The surface temperatures may be evaluated from Eqs. 8.68 and 8.69 with

$$Re_D = \frac{r \; u_m D_h}{m} = \frac{\dot{m}_a \; (D_o - D_i)}{(p/4) \; \left(D_o^2 - D_i^2\right) m} = \frac{4 \; \dot{m}_a}{p \; (D_o + D_i) m} = \frac{4(2.1 \text{ kg/s})}{p \; (4.05m) \; 270 \times 10^{-7} N \cdot s/m^2}$$

$$Re_D = 24,452$$

the flow is turbulent and from Eq. 8.60

$$h_i \approx h_o \approx \frac{k}{D_h} 0.023 \; Re_D^{4/5} \; Pr^{0.4} = \frac{0.041 \text{ W/m} \cdot K}{0.05 \text{ m}} 0.023 (24,452)^{4/5} (0.68)^{0.4} = 52 \text{ W/m}^2 \cdot K.$$

With $\quad q_i'' = q_i' / p \; D_i = 1.25 \times 10^5 W/m / p \times 2m = 19,900 \text{ W/m}^2$

Eq. 8.68 gives

$$\left(T_{s,i} - T_m\right) = q_i'' / h_i = 19,900 \text{ W/m}^2 / 52 \text{ W/m}^2 \cdot K = 383K$$

$$T_{s,i,1} = 683K \qquad T_{s,i,2} = 1087K.$$ <

From Eq. 8.69, with $q_o'' = 0$, $\left(T_{s,o} - T_m\right) = 0$. Hence

$$T_{s,o,1} = T_{a,1} = 300K.$$ <