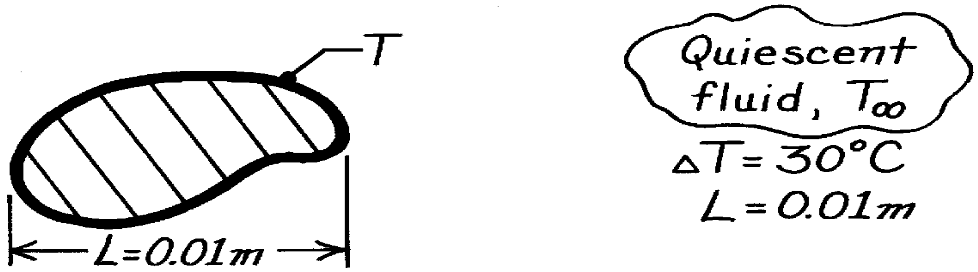
PROBLEM 9.3

KNOWN: Relation for the Rayleigh number.

FIND: Rayleigh number for four fluids for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Perfect gas behavior for specified gases.

PROPERTIES: *Table A-4*, Air (400K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$, $\alpha = 38.3 \times 10^{-6}\ m^2/s$, $\beta = 1/T = 1/400K = 2.50 \times 10^{-3}\ K^{-1}$; *Table A-4*, Helium (400K, 1 atm): $\nu = 199 \times 10^{-6}\ m^2/s$, $\alpha = 295 \times 10^{-6}\ m^2/s$, $\beta = 1/T = 2.50 \times 10^{-3}\ K^{-1}$; *Table A-5*, Glycerin (12°C = 285K): $\nu = 2830 \times 10^{-6}\ m^2/s$, $\alpha = 0.964 \times 10^{-7}\ m^2/s$, $\beta = 0.475 \times 10^{-3}\ K^{-1}$; *Table A-6*, Water (37°C = 310K, sat. liq.): $\nu = \mu_f\, v_f = 695 \times 10^{-6}\ N \cdot s/m^2 \times 1.007 \times 10^{-3}\ m^3/kg = 0.700 \times 10^{-6}\ m^2/s$, $\alpha = k_f\, v_f/c_{p,f} = 0.628\ W/m \cdot K \times 1.007 \times 10^{-3}\ m^3/kg/4178\ J/kg \cdot K = 0.151 \times 10^{-6}\ m^2/s$, $\beta_f = 361.9 \times 10^{-6}\ K^{-1}$.

ANALYSIS: The Rayleigh number, a dimensionless parameter used in free convection analysis, is defined as the product of the Grashof and Prandtl numbers.

$$Ra_L \equiv Gr \cdot Pr = \frac{g b \Delta T L^3}{n^2}\ \frac{m c_p}{k} = \frac{g b \Delta T L^3}{n^2} \cdot \frac{(nr)\, c_p}{k} = \frac{g b \Delta T L^3}{na}$$

where $\alpha = k/\rho c_p$ and $\nu = \mu/\rho$. The numerical values for the four fluids follow:

Air (400K, 1 atm)

$$Ra_L = 9.8 m/s^2\ (1/400K)\ 30K\,(0.01m)^3 / 26.41 \times 10^{-6}\ m^2/s \times 38.3 \times 10^{-6}\ m^2/s = 727 \quad <$$

Helium (400K, 1 atm)

$$Ra_L = 9.8 m/s^2\,(1/400K)\ 30K\,(0.01m)^3 / 199 \times 10^{-6} m^2/s \times 295 \times 10^{-6} m^2/s = 12.5 \quad <$$

Glycerin (285K)

$$Ra_L = 9.8 m/s^2 \left(0.475 \times 10^{-3} K^{-1}\right) 30K\,(0.01m)^3 / 2830 \times 10^{-6} m^2/s \times 0.964 \times 10^{-7} m^2/s = 512 \quad <$$

Water (310K)

$$Ra_L = 9.8 m/s^2 \left(0.362 \times 10^{-3} K^{-1}\right) 30K\,(0.01m)^3 / 0.700 \times 10^{-6} m^2/s \times 0.151 \times 10^{-6} m^2/s = 9.35 \times 10^5. \quad <$$

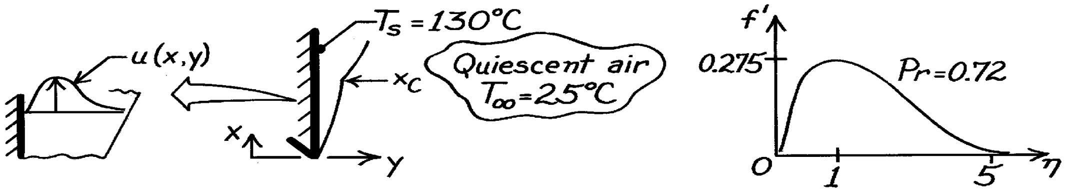
COMMENTS: (1) Note the wide variation in values of Ra for the four fluids. A large value of Ra implies enhanced free convection, however, other properties affect the value of the heat transfer coefficient.

PROBLEM 9.6

KNOWN: Large vertical plate with uniform surface temperature of 130°C suspended in quiescent air at 25°C and atmospheric pressure.

FIND: (a) Boundary layer thickness at 0.25 m from lower edge, (b) Maximum velocity in boundary layer at this location and position of maximum, (c) Heat transfer coefficient at this location, (d) Location where boundary layer becomes turbulent.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal, vertical surface in an extensive, quiescent medium, (2) Boundary layer assumptions valid.

PROPERTIES: *Table A-4*, Air $\left(T_f = \left(T_s + T_\infty\right)/2 = 350\text{K, 1 atm}\right)$: $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) From the similarity solution results, Fig. 9.4 (see above right), the boundary layer thickness corresponds to a value of $\eta \approx 5$. From Eqs. 9.13 and 9.12,

$$y = \eta\, x \left(Gr_x / 4\right)^{-1/4} \qquad (1)$$

$$Gr_x = g\beta\left(T_s - T_\infty\right)x^3/\nu^2 = 9.8\frac{m}{s^2}\times\frac{1}{350K}\left(130-25\right)K\,x^3/\left(20.92\times10^{-6}m^2/s\right)^2 = 6.718\times10^9\,x^3 \qquad (2)$$

$$y \approx 5\left(0.25m\right)\left(6.718\times10^9\left(0.25\right)^3/4\right)^{-1/4} = 1.746\times10^{-2}m = 17.5\text{ mm.} \qquad (3) \quad <$$

(b) From the similarity solution shown above, the maximum velocity occurs at $\eta \approx 1$ with $f'(\eta) = 0.275$. From Eq.9.15, find

$$u = \frac{2\nu}{x}Gr_x^{1/2}\,f'(\eta) = \frac{2\times20.92\times10^{-6}m^2/s}{0.25m}\left(6.718\times10^9\left(0.25\right)^3\right)^{1/2}\times0.275 = 0.47\text{ m/s.} \quad <$$

The maximum velocity occurs at a value of η = 1; using Eq. (3), it follows that this corresponds to a position in the boundary layer given as

$$y_{max} = 1/5\ \left(17.5\text{ mm}\right) = 3.5\text{ mm.} \quad <$$

(c) From Eq. 9.19, the local heat transfer coefficient at x = 0.25 m is

$$Nu_x = h_x x/k = \left(Gr_x/4\right)^{1/4}\ g\left(Pr\right) = \left(6.718\times10^9\left(0.25\right)^3/4\right)^{1/4} 0.586 = 41.9$$

$$h_x = Nu_x k/x = 41.9\times0.030\text{ W/m}\cdot\text{K}/0.25\text{ m} = 5.0\text{ W/m}^2\cdot\text{K.} \quad <$$

The value for g(Pr) is determined from Eq. 9.20 with Pr = 0.700.

(d) According to Eq. 9.23, the boundary layer becomes turbulent at x_c given as

$$Ra_{x,c} = Gr_{x,c}\,Pr \approx 10^9 \qquad x_c \approx \left[10^9/6.718\times10^9\left(0.700\right)\right]^{1/3} = 0.60\text{ m.} \quad <$$

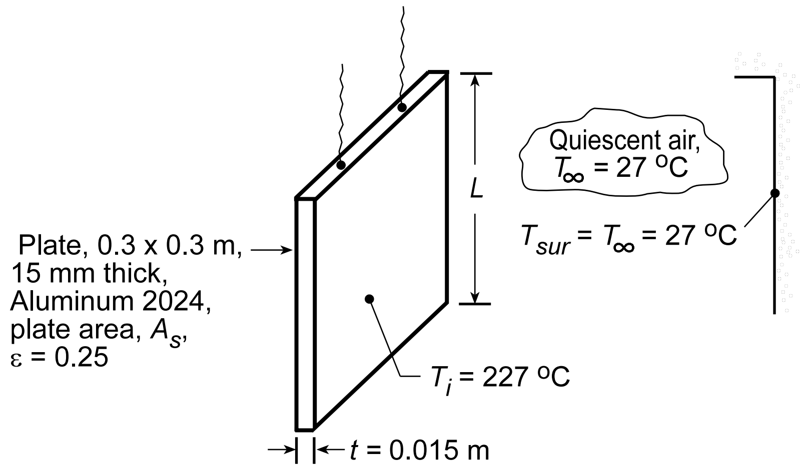
COMMENTS: Note that $\beta = 1/T_f$ is a suitable approximation for air.

PROBLEM 9.14

KNOWN: Aluminum plate (alloy 2024) at an initial uniform temperature of 227°C is suspended in a room where the ambient air and surroundings are at 27°C.

FIND: (a) Expression for time rate of change of the plate, (b) Initial rate of cooling (K/s) when plate temperature is 227°C, (c) Validity of assuming a uniform plate temperature, (d) Decay of plate temperature and the convection and radiation rates during cooldown.

SCHEMATIC:



ASSUMPTIONS: (1) Plate temperature is uniform, (2) Ambient air is quiescent and extensive, (3) Surroundings are large compared to plate.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 (T = 500 K): $\rho = 2770\ \text{kg/m}^3$, k = 186 W/m·K, c = 983 J/kg·K; *Table A.4*, Air (T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}\ \text{m}^2\text{/s}$, k = 0.0388 W/m·K, $\alpha = 38.3 \times 10^{-6}\ \text{m}^2\text{/s}$, Pr = 0.690.

ANALYSIS: (a) From an energy balance on the plate with free convection and radiation exchange, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain

$$-\bar{h}_L 2A_s\left(T_s - T_\infty\right) - \varepsilon 2A_s\sigma\left(T_s^4 - T_{sur}^4\right) = \rho A_s tc\frac{dT}{dt} \quad \text{or} \quad \frac{dT}{dt} = \frac{-2}{\rho tc}\left[\bar{h}_L\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] \quad <$$

where T_s, the plate temperature, is assumed to be uniform at any time.

(b) To evaluate (dT/dt), estimate $\bar{h}_L$. First, find the Rayleigh number,

$$Ra_L = g\beta\left(T_s - T_\infty\right)L^3/\nu\alpha = \frac{9.8\,\text{m/s}^2\,(1/400\,\text{K})(227-27)\text{K}\times(0.3\,\text{m})^3}{26.41\times10^{-6}\,\text{m}^2/\text{s}\times38.3\times10^{-6}\,\text{m}^2/\text{s}} = 1.308\times10^8.$$

Eq. 9.27 is appropriate; substituting numerical values, find

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.308\times10^8\right)^{1/4}}{\left[1+\left(0.492/0.690\right)^{9/16}\right]^{4/9}} = 55.5$$

$$\bar{h}_L = \overline{Nu}_L k/L = 55.5\times0.0338\,\text{W/m}\cdot\text{K}/0.3\,\text{m} = 6.25\,\text{W/m}^2\cdot\text{K}$$

Continued...

PROBLEM 9.14 (Cont.)

$$\frac{dT}{dt} = \frac{-2}{2770\,\text{kg/m}^3 \times 0.015\,\text{m} \times 983\,\text{J/kg}\cdot\text{K}} \times$$

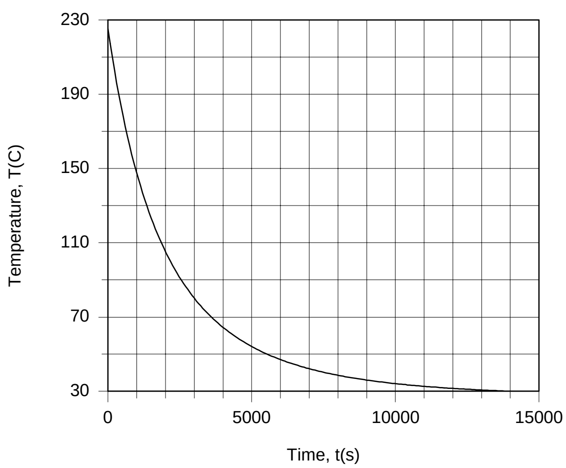
$$\left[6.25\,\text{W/m}^2\cdot\text{K}\,(227-27)\,\text{K} + 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(500^4 - 300^4\right)\text{K}^4\right] = -0.099\,\text{K/s}\,.$$ <

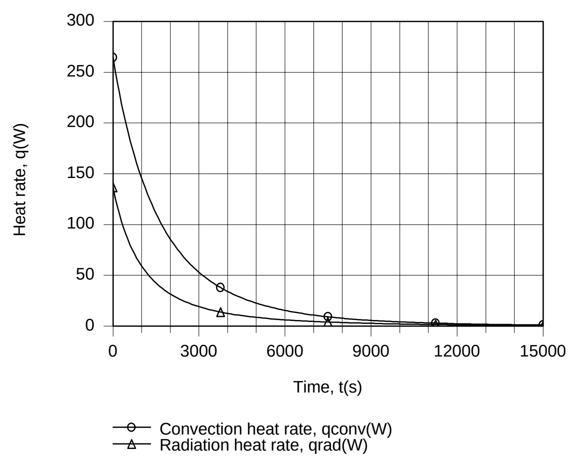
(c) The uniform temperature assumption is justified if the Biot number criterion is satisfied. With $L_c \equiv (V/2A_s) = (A_s \cdot t/2A_s) = (t/2)$ and $\overline{h}_{tot} = \overline{h}_{conv} + \overline{h}_{rad}$, Bi = $\overline{h}_{tot}(t/2)/k \le 0.1$. Using the linearized radiation coefficient relation, find

$$\overline{h}_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)(500+300)\left(500^2+300^2\right)\text{K}^3 = 3.86\,\text{W/m}^2\cdot\text{K}$$

Hence, Bi = (6.25 + 3.86) $\text{W/m}^2\cdot\text{K}$(0.015 m/2)/186 W/m·K = 4.07×10^{-4}. Since Bi << 0.1, the assumption is appropriate.

(d) The temperature history of the plate was computed by combining the *Lumped Capacitance Model* of IHT with the appropriate *Correlations* and *Properties* Toolpads.





Due to the small values of $\overline{h}_L$ and $\overline{h}_{rad}$, the plate cools slowly and does not reach 30°C until t ≈ 14000s = 3.89h. The convection and radiation rates decrease rapidly with increasing t (decreasing T), thereby decelerating the cooling process.

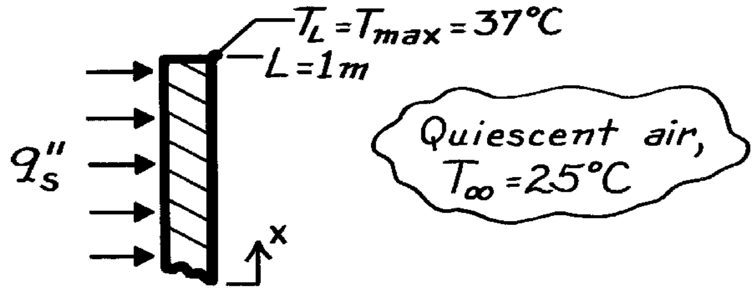
COMMENTS: The reduction in the convection rate with increasing time is due to a reduction in the thermal conductivity of air, as well as the values of $\overline{h}_L$ and T.

PROBLEM 9.26

KNOWN: Vertical panel with uniform heat flux exposed to ambient air.

FIND: Allowable heat flux if maximum temperature is not to exceed a specified value, T_{max}.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Radiative exchange with surroundings negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_{L/2} + T_\infty)/2 = (35.4 + 25)°C/2 = 30.2°C = 303K$, 1 atm): $\nu = 16.19 \times 10^{-6}$ m²/s, $k = 26.5 \times 10^{-3}$ W/m·K, $\alpha = 22.9 \times 10^{-6}$ m²/s, Pr = 0.707.

ANALYSIS: Following the treatment of Section 9.6.1 for a vertical plate with uniform heat flux (constant q_s''), the heat flux can be evaluated as

$$q_s'' = \bar{h}\Delta T_{L/2} \qquad \text{where} \qquad \Delta T_{L/2} = T_s(L/2) - T_\infty \qquad (1,2)$$

and $\bar{h}$ is evaluated using an appropriate correlation for a constant temperature vertical plate. From Eq. 9.28,

$$\Delta T_x \equiv T_x - T_\infty = 1.15(x/L)^{1/5}\Delta T_{L/2} \qquad (3)$$

and recognizing that the maximum temperature will occur at the top edge, x = L, use Eq. (3) to find

$$\Delta T_{L/2} = (37-25)^\circ C/1.15(1/1)^{1/5} = 10.4^\circ C \qquad \text{or} \qquad T_{L/2} = 35.4^\circ C.$$

Calculate now the Rayleigh number based upon $\Delta T_{L/2}$, with $T_f = (T_{L/2} + T_\infty)/2 = 303K$,

$$Ra_L = \frac{g b \Delta T L^3}{n a} \qquad \text{where} \qquad \Delta T = \Delta T_{L/2} \qquad (4)$$

$$Ra_L = 9.8 m/s^2 (1/303K) \times 10.4K(1m)^3 / 16.19 \times 10^{-6} m^2/s \times 22.9 \times 10^{-6} m^2/s = 9.07 \times 10^8.$$

Since $Ra_L < 10^9$, the boundary layer flow is laminar; hence the correlation of Eq. 9.27 is appropriate,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} \qquad (5)$$

$$\bar{h} = \left[\frac{0.0265 W/m \cdot K}{1m}\right]\left\{0.68 + 0.670\left(9.07 \times 10^8\right)^{1/4} / \left[1 + (0.492/0.707)^{9/16}\right]^{4/9}\right\} = 2.38 W/m^2 \cdot K.$$

From Eqs. (1) and (2) with numerical values for $\bar{h}$ and $\Delta T_{L/2}$, find

$$q_s'' = 2.38 W/m^2 \cdot K \times 10.4^\circ C = 24.8 W/m^2. \qquad <$$

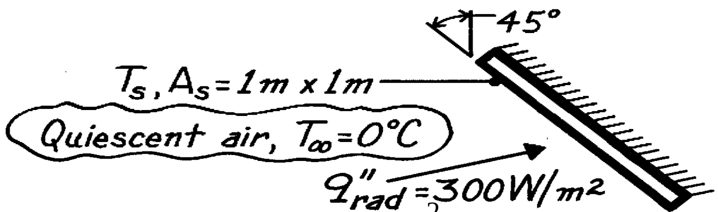
COMMENTS: Recognize that radiation exchange with the environment will be significant. Assuming $\bar{T}_s = T_{L/2}$, $T_{sur} = T_\infty$ and $\varepsilon = 1$, find $q_{rad}'' = s\left(\bar{T}_s^{-4} - T_{sur}^4\right) = 66 W/m^2$.

PROBLEM 9.52

KNOWN: Plate, 1m × 1m, inclined at 45° from the vertical is exposed to a net radiation heat flux of 300 W/m^2; backside of plate is insulated and ambient air is at 0°C.

FIND: Temperature plate reaches for the prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Net radiation heat flux (300 W/m^2) includes exchange with surroundings, (2) Ambient air is quiescent, (3) No heat losses from backside of plate, (4) Steady-state conditions.

PROPERTIES: *Table A-4*, Air (assuming T_s = 84°C, $T_f = (T_s + T_\infty)/2$ = (84 + 0)°C/2 = 315K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705, $\beta = 1/T_f$.

ANALYSIS: From an energy balance on the plate, it follows that $q''_{rad} = q''_{conv}$. That is, the net radiation heat flux into the plate is equal to the free convection heat flux to the ambient air. The temperature of the surface can be expressed as

$$T_s = T_\infty + q''_{rad} / \overline{h}_L \qquad (1)$$

where $\overline{h}_L$ must be evaluated from an appropriate correlation. Since this is the *bottom surface of a heated inclined plate*, "*g*" may be replaced by "*g cos q*"; hence using Eq. 9.25, find

$$Ra_L = \frac{g\cos q b\,(T_s - T_\infty) L^3}{na} = \frac{9.8\,\text{m/s}^2 \times \cos 45° (1/315\,\text{K})(84-0)\,\text{K}\,(1\text{m})^3}{17.40\times10^{-6}\,\text{m}^2/\text{s} \times 24.7\times10^{-6}\,\text{m}^2/\text{s}} = 4.30\times10^9.$$

Since $Ra_L > 10^9$, conditions are turbulent and Eq. 9.26 is appropriate for estimating $\overline{Nu}_L$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+(0.492/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 \qquad (2)$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(4.30\times10^9\right)^{1/6}}{\left[1+(0.492/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 193.2$$

$$\overline{h}_L = \overline{Nu}_L\, k/L = 193.2 \times 0.0274\,\text{W/m}\cdot\text{K}/1\text{m} = 5.29\,\text{W/m}^2\cdot\text{K}. \qquad (3)$$

Substituting $\overline{h}_L$ from Eq. (3) into Eq. (1), the plate temperature is

$$T_s = 0°\text{C} + 300\,\text{W/m}^2 / 5.29\,\text{W/m}^2\cdot\text{K} = 57°\text{C}.$$ <

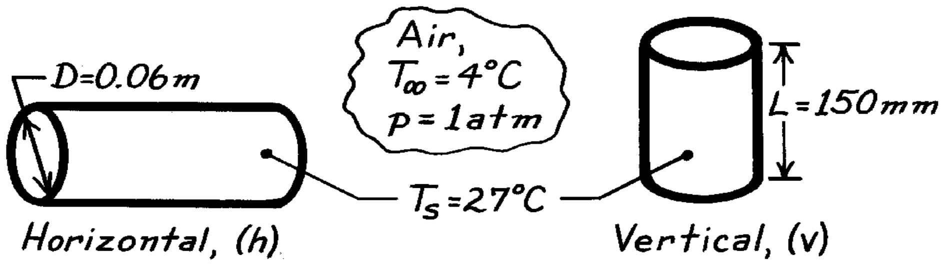
COMMENTS: Note that the resulting value of $T_s \approx 57$°C is substantially lower than the assumed value of 84°C. The calculation should be repeated with a new estimate of T_s, say, 60°C. An alternate approach is to write Eq. (2) in terms of T_s, the unknown surface temperature and then combine with Eq. (1) to obtain an expression which can be solved, by trial-and-error, for T_s.

PROBLEM 9.56

KNOWN: Dimensions and temperature of beer can in refrigerator compartment.

FIND: Orientation which maximizes cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) End effects are negligible, (2) Compartment air is quiescent, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = 288.5K, 1 atm): ν = 14.87 × 10^{-6} m^2/s, k =0.0254 W/m·K, α = 21.0 × 10^{-6} m^2/s, Pr = 0.71, β = 1/T_f = 3.47 × 10^{-3} K^{-1}.

ANALYSIS: The ratio of cooling rates may be expressed as

$$\frac{q_v}{q_h} = \frac{\overline{h}_v}{\overline{h}_h}\frac{p\,DL}{p\,DL}\frac{(T_s - T_\infty)}{(T_s - T_\infty)} = \frac{\overline{h}_v}{\overline{h}_h}.$$

For the *vertical* surface, find

$$Ra_L = \frac{g b\,(T_s - T_\infty)}{n a}L^3 = \frac{9.8\,m/s^2 \times 3.47\times10^{-3}\,K^{-1}\,(23°C)}{\left(14.87\times10^{-6} m^2/s\right)\left(21\times10^{-6} m^2/s\right)}L^3 = 2.5\times10^9 L^3$$

$$Ra_L = 2.5\times10^9 (0.15)^3 = 8.44\times10^6,$$

and using the correlation of Eq. 9.26, $$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(8.44\times10^6\right)^{1/6}}{\left[1+(0.492/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 29.7.$$

Hence $$\overline{h}_L = \overline{h}_v = \overline{Nu}_L\frac{k}{L} = 29.7\frac{0.0254\,W/m\cdot K}{0.15m} = 5.03\,W/m^2\cdot K.$$

For the *horizontal* surface, find $Ra_D = \frac{g b\,(T_s - T_\infty)}{n a}D^3 = 2.5\times10^9 (0.06)^3 = 5.4\times10^5$

and using the correlation of Eq. 9.34, $$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(5.4\times10^5\right)^{1/6}}{\left[1+(0.559/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 12.24$$

$$\overline{h}_D = \overline{h}_h = \overline{Nu}_D\frac{k}{D} = 12.24\frac{0.0254\,W/m\cdot K}{0.06m} = 5.18\,W/m^2\cdot K.$$

Hence $$\frac{q_v}{q_h} = \frac{5.03}{5.18} = 0.97.$$ <

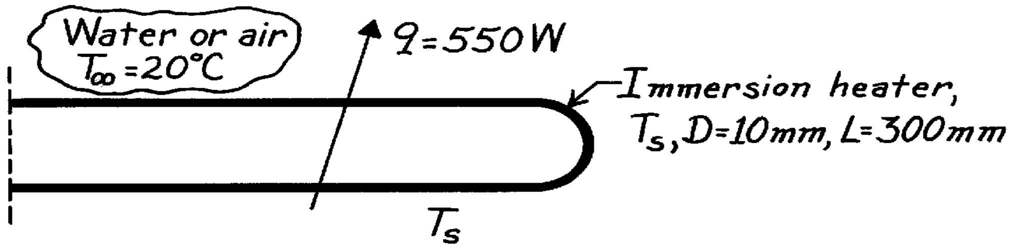
COMMENTS: In view of the uncertainties associated with Eqs. 9.26 and 9.34 and the neglect of end effects, the above result is inconclusive. The cooling rates are approximately the same.

PROBLEM 9.62

KNOWN: Dissipation rate of an immersion heater in a large tank of water.

FIND: Surface temperature in water and, if accidentally operated, in air.

SCHEMATIC:



ASSUMPTIONS: (1) Quiescent ambient fluid, (2) Negligible radiative exchange.

PROPERTIES: *Table A-6*, Water and *Table A-4*, Air:

	T(K)	$k \cdot 10^3$(W/m·K)	$\nu \cdot 10^7$(μ/ρ,m^2/s)	$\alpha \cdot 10^7$($k/\rho c_p$,m^2/s)	Pr	$\beta \cdot 10^6$(K^{-1})
Water	315	634	6.25	1.531	4.16	400.4
Air	1500	100	2400	3500	0685	666.7

ANALYSIS: From the rate equation, the surface temperature, T_s, is

$$T_s = T_\infty + q / \left(p D L \bar{h} \right) \qquad (1)$$

where $\bar{h}$ is estimated by an appropriate correlation. Since such a calculation requires knowledge of T_s, an iteration procedure is required. Begin by assuming for *water* that T_s = 64°C such that T_f = 315K. Calculate the Rayleigh number,

$$Ra_D = \frac{g b \Delta T D^3}{n a} = \frac{9.8 m/s^2 \times 400.4 \times 10^{-6} K^{-1} (64 - 20) K (0.010 m)^3}{6.25 \times 10^{-7} m^2/s \times 1.531 \times 10^{-7} m^2/s} = 1.804 \times 10^6. \qquad (2)$$

Using the Churchill-Chu relation, find

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16} \right]^{8/27}} \right\}^2 \qquad (3)$$

$$\bar{h} = \frac{0.634 W/m \cdot K}{0.01 m} \left\{ 0.60 + \frac{0.387 \left(1.804 \times 10^6 \right)^{1/6}}{\left[1 + (0.559/4.16)^{9/16} \right]^{8/27}} \right\}^2 = 1301 W/m^2 \cdot K.$$

Substituting numerical values into Eq. (1), the calculated value for T_s in *water* is

$$T_s = 20°C + 550 W / p \times 0.010 m \times 0.30 m \times 1301 W/m^2 \cdot K = 64.8°C.$$ <

Continued …..

Our initial assumption of $T_s = 64°C$ is in excellent agreement with the calculated value.

With accidental operation in *air*, the heat transfer coefficient will be nearly a factor of 100 less. Suppose $\bar{h} \approx 25 W/m^2 \cdot K$, then from Eq. (1), $T_s \approx 2360°C$. Very likely the heater will burn out. Using air properties at $T_f \approx 1500K$ and Eq. (2), find $Ra_D = 1.815 \times 10^2$. Using Eq. 9.33, $Nu_D = CRa_D^n$ with C= 0.85 and n = 0.188 from Table 9.1, find $\bar{h} = 22.6 W/m^2 \cdot K$. Hence, our first estimate for the surface temperature in *air* was reasonable,

$$T_s \approx 2300°C. \qquad <$$

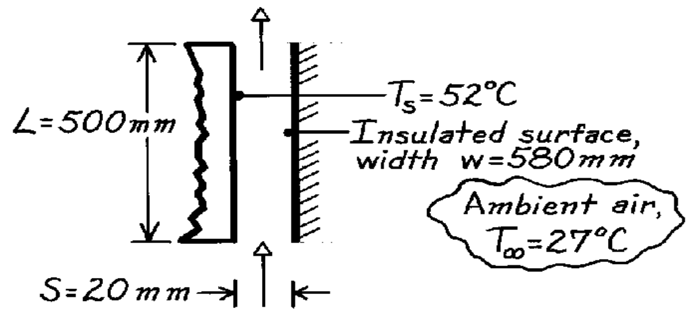
However, radiation exchange will be the dominant mode, and would reduce the estimate for T_s. Generally such heaters could not withstand operating temperatures above 1000°C and safe operation in air is not possible.

PROBLEM 9.85

KNOWN: Vertical air vent in front door of dishwasher with prescribed width and height. Spacing between isothermal and insulated surface of 20 mm.

FIND: (a) Heat loss from the tub surface and (b) Effect on heat rate of changing spacing by ± 10 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vent forms vertical parallel isothermal/adiabatic plates, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, ($T_f = (T_s + T_\infty)/2$ = 312.5K, 1 atm): $\nu = 17.15 \times 10^{-6}$ m²/s, $\alpha = 24.4 \times 10^{-6}$ m²/s, $k = 27.2 \times 10^{-3}$ W/m·K, $\beta = 1/T_f$.

ANALYSIS: The vent arrangement forms two vertical plates, one is isothermal, T_s, and the other is adiabatic $(q'' = 0)$. The heat loss can be estimated from Eq. 9.37 with the correlation of Eq. 9.45 using C_1 = 144 and C_2 = 2.87 from Table 9.3:

$$Ra_S = \frac{g b\,(T_s - T_\infty)S^3}{n a} = \frac{9.8\,\text{m/s}^2\,(1/312.5\text{ K})(52-27)\text{K}\,(0.020\text{m})^3}{17.15\times10^{-6}\text{m}^2/\text{s}\times24.4\times10^{-6}\text{m}^2/\text{s}} = 14{,}988$$

$$q = A_s\,(T_s - T_\infty)\frac{k}{S}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = (0.500\times0.580)\text{m}^2\times$$

$$(52-27)\text{K}\,\frac{0.0272\,\text{W/m}\cdot\text{K}}{0.020\text{m}}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = 28.8\text{W}. \quad <$$

(b) To determine the effect of the spacing at S = 30 and 10 mm, we need only repeat the above calculations with these results

S (mm)	Ra_S	q (W)	
10	1874	26.1	<
30	50,585	28.8	<

Since it would be desirable to minimize heat losses from the tub, based upon these calculations, you would recommend a decrease in the spacing.

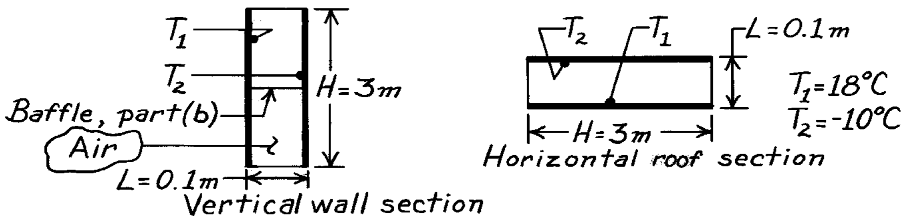
COMMENTS: For this situation, according to Table 9.3, the spacing corresponding to the maximum heat transfer rate is $S_{max} = (S_{max}/S_{opt}) \times 2.15(Ra_S/S^3L)^{-1/4}$ = 14.5 mm. Find q_{max} = 28.5 W. Note that the heat rate is not very sensitive to spacing for these conditions.

PROBLEM 9.94

KNOWN: Horizontal flat roof and vertical wall sections of same dimensions exposed to identical temperature differences.

FIND: (a) Ratio of convection heat rate for horizontal section to that of the vertical section and (b) Effect of inserting a baffle at the mid-height of the vertical wall section on the convection heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Ends of sections and baffle adiabatic, (2) Steady-state conditions.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (T_1 + T_2)/2 = 277\text{K}, 1\text{atm}\right)$: $\nu = 13.84 \times 10^{-6}$ m²/s, k = 0.0245 W/m·K, $\alpha = 19.5 \times 10^{-6}$ m²/s, Pr = 0.713.

ANALYSIS: (a) The ratio of the convection heat rates is

$$\frac{q_{hor}}{q_{vert}} = \frac{\overline{h}_{hor} A_s \Delta T}{\overline{h}_{vert} A_s \Delta T} = \frac{\overline{h}_{hor}}{\overline{h}_{vert}}. \quad (1)$$

To estimate coefficients, recognizing both sections have the same characteristics length, L = 0.1m, with $Ra_L = g\beta\Delta TL^3/\nu\alpha$ find

$$Ra_L = \frac{9.8\,\text{m/s}^2 \times (1/277\text{K})(18-(-10))\text{K}(0.1\text{m})^3}{13.84\times10^{-6}\text{m}^2/\text{s}\times19.5\times10^{-6}\text{m}^2/\text{s}} = 3.67\times10^6.$$

The appropriate correlations for the sections are Eqs. 9.49 and 9.52 (with H/L = 30),

$$\overline{Nu}_L\Big|_{hor} = 0.069 Ra_L^{1/3}\,Pr^{0.074} \qquad \overline{Nu}_L\Big|_{vert} = 0.42 Ra_L^{1/4}\,Pr^{0.012}\,(H/L)^{-0.3}. \quad (3,4)$$

Using Eqs. (3) and (4), the ratio of Eq. (1) becomes,

$$\frac{q_{hor}}{q_{vert}} = \frac{0.069 Ra_L^{1/3}\,Pr^{0.074}}{0.42 Ra_L^{1/4}\,Pr^{0.012}\,(H/L)^{-0.3}} = \frac{0.069\left(3.67\times10^6\right)^{1/3}(0.713)^{0.074}}{0.42\left(3.67\times10^6\right)^{1/4}(0.713)^{0.012}(30)^{-0.3}} = 1.57. \quad <$$

(b) The effect of the baffle in the vertical wall section is to reduce H/L from 30 to 15. Using Eq. 9.52, it follows,

$$\frac{q_{baf}}{q} = \frac{\overline{h}_{baf}}{\overline{h}} = \frac{(H/L)_{baf}^{-0.3}}{(H/L)^{-0.3}} = \left(\frac{15}{30}\right)^{-0.3} = 1.23. \quad <$$

That is, the effect of the baffle is to increase the convection heat rate.

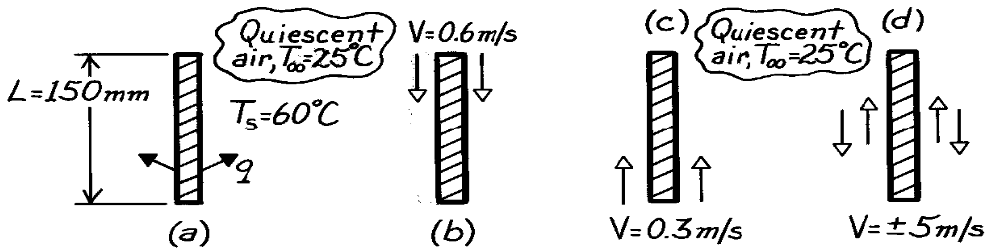
COMMENTS: (1) Note that the heat rate for the horizontal section is 57% larger than that for the vertical section for the same $(T_1 - T_2)$. This indicates the importance of heat losses from the ceiling or roofs in house construction. (2) Recognize that for Eq. 9.52, the Pr > 1 requirement is not completely satisfied. (3) What is the physical explanation for the result of part (b)?

PROBLEM 9.109

KNOWN: Vertical array of circuit boards 0.15m high with maximum allowable uniform surface temperature for prescribed ambient air temperature.

FIND: Allowable electrical power dissipation per board, $q'[W/m]$, for these cooling arrangements: (a) Free convection only, (b) Air flow downward at 0.6 m/s, (c) Air flow upward at 0.3 m/s, and (d) Air flow upward or downward at 5 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Board horizontal spacing sufficient that boundary layers don't interfere, (3) Ambient air behaves as quiescent medium, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 \approx 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}\ m^2/s$, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}\ m^2/s$, Pr = 0.705, $\beta = 1/T_f$.

ANALYSIS: (a) For *free convection* only, the allowable electrical power dissipation rate is

$$q' = \bar{h}_L (2L)(T_s - T_\infty) \qquad (1)$$

where $\bar{h}_L$ is estimated using the appropriate correlation for free convection from a vertical plate. Find the Rayleigh number,

$$Ra_L = \frac{g\,b\,\Delta T\,L^3}{na} = \frac{9.8 m/s^2 (1/315K)(60-25)K(0.150m)^3}{17.4\times10^{-6} m^2/s \times 24.7\times10^{-6} m^2/s} = 8.551\times10^6. \qquad (2)$$

Since $Ra_L < 10^9$, the flow is laminar. With Eq. 9.27 find

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{\left(0.670\left[8.551\times10^6\right]^{1/4}\right)}{\left[1+(0.492/0.705)^{9/16}\right]^{4/9}} = 28.47 \qquad (3)$$

$$\bar{h}_L = (0.0274 W/m\cdot K/0.150m)\times 28.47 = 5.20 W/m^2\cdot K.$$

Hence, the allowable electrical power dissipation rate is,

$$q' = 5.20 W/m^2\cdot K(2\times0.150m)(60-25)°C = 54.6 W/m.$$ <

(b) With *downward velocity* V = 0.6 m/s, the possibility of mixed forced-free convection must be considered. With $Re_L = VL/\nu$, find

$$\left(Gr_L/Re_L^2\right) = \left(\frac{Ra_L}{Pr}/Re_L^2\right) \qquad (4)$$

$$\left(Gr_L/Re_L^2\right) = \left(8.551\times10^6/0.705\right)/\left(0.6 m/s\times0.150m/17.40\times10^{-6} m^2/s\right)^2 = 0.453.$$

Continued …..

Since $\left(Gr_L / Re_L^2\right) \sim 1$, flow is mixed and the average heat transfer coefficient may be found from a correlating equation of the form

$$\overline{Nu}^n = Nu_F^n \pm Nu_N^n \tag{5}$$

where n = 3 for the vertical plate geometry and the minus sign is appropriate since the natural convection (N) flow opposes the forced convection (F) flow. For the forced convection flow, Re_L = 5172 and the flow is laminar; using Eq. 7.31,

$$\overline{Nu}_F = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(5172)^{1/2}(0.705)^{1/3} = 42.50. \tag{6}$$

Using $\overline{Nu}_N = 28.47$ from Eq. (3), Eq. (5) now becomes

$$\overline{Nu}^3 = \left(\frac{\bar{h}L}{k}\right)^3 = (42.50)^3 - (28.47)^3 \qquad \overline{Nu} = 37.72$$

$$\bar{h} = \left(\frac{0.0274\,W/m\cdot K}{0.150m}\right) \times 37.72 = 6.89\,W/m^2\cdot K.$$

Substituting for $\bar{h}$ into the rate equation, Eq. (1), the allowable power dissipation with a downward velocity of 0.6 m/s is

$$q' = 6.89\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 72.3\,W/m. \qquad <$$

(c) With an *upward velocity* V = 0.3 m/s, the positive sign of Eq. (5) applies since the N-flow is assisting the F-flow. For forced convection, find

$$Re_L = VL/n = 0.3m/s \times 0.150m/\left(17.40\times 10^{-6} m^2/s\right) = 2586.$$

The flow is again laminar, hence Eq. (6) is appropriate.

$$\overline{Nu}_F = 0.664(2586)^{1/2}(0.705)^{1/3} = 30.05.$$

From Eq. (5), with the positive sign, and $\overline{Nu}_N$ from Eq. (4),

$$\overline{Nu}^3 = (30.05)^3 + (28.47)^3 \quad \text{or} \quad \overline{Nu} = 36.88 \quad \text{and} \quad \bar{h} = 6.74\,W/m^2\cdot K.$$

From Eq. (1), the allowable power dissipation with an upward velocity of 0.3 m/s is

$$q' = 6.74\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 70.7\,W/m. \qquad <$$

(d) With a *forced convection* velocity V = 5 m/s, very likely forced convection will dominate. Check by evaluating whether $\left(Gr_L\ Re_L^2\right) << 1$ where $Re_L = VL/\nu$ = 5 m/s × 0.150m/(17.40 × 10^{-6} m^2/s) = 43,103. Hence,

$$\left(Gr_L / Re_L^2\right) = \left(\frac{Ra_L}{Pr} / Re_L^2\right) = \left(8.551\times 10^6 / 0.705\right)/43{,}103^2 = 0.007.$$

The flow is not mixed, but pure forced convection. Using Eq. (6), find

$$\bar{h} = (0.0274\,W/m\cdot K/0.150m)\,0.664(43{,}103)^{1/2}(0.705)^{1/3} = 22.4\,W/m^2\cdot K$$

and the allowable dissipation rate is

$$q' = 22.4\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 235\,W/m. \qquad <$$

COMMENTS: Be sure to compare dissipation rates to see relative importance of mixed flow conditions.