PROBLEM 13.1

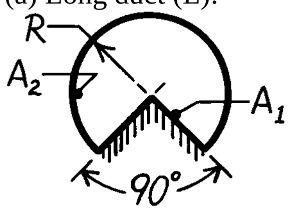
KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

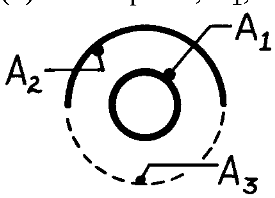
(a) Long duct (L):



By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \dfrac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

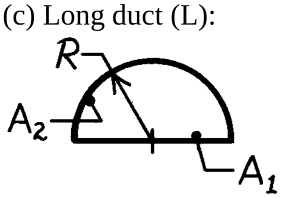


Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):



<

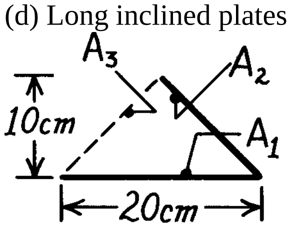
By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{2RL}{\pi RL} \times 1.0 = \dfrac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

By inspection,

$F_{12} = 1.0$

(d) Long inclined plates (L):

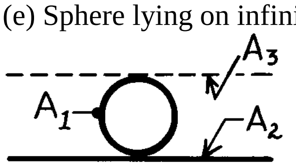


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} = \dfrac{20L}{10(2)^{1/2}\,L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane



Summation rule, $F_{11} + F_{12} + F_{13} = 1$

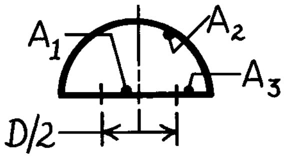
But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \dfrac{A_1}{A_2} F_{12} \rightarrow 0$ since $A_2 \rightarrow \infty.$ <

Continued …..

PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.



By inspection, $F_{12} = 1.0$ <

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence, } F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

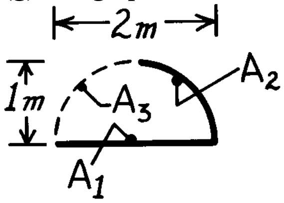
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[\frac{D}{2} \right]^2 / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$ <

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):



Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

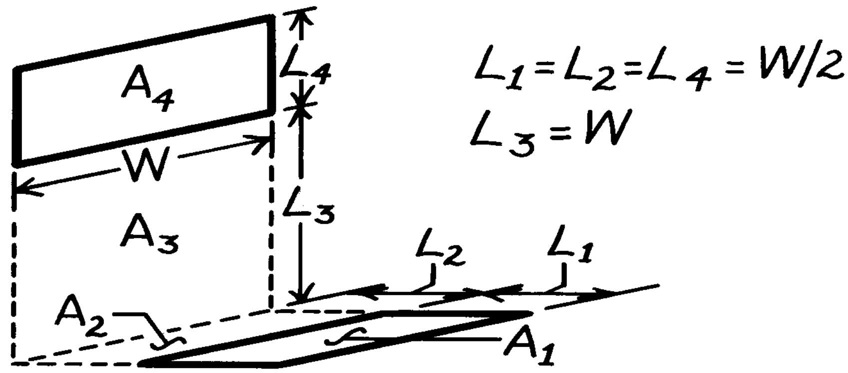
(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

PROBLEM 13.10

KNOWN: Arrangement of perpendicular surfaces without a common edge.

FIND: (a) A relation for the view factor F_{14} and (b) The value of F_{14} for prescribed dimensions.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces.

ANALYSIS: (a) To determine F_{14}, it is convenient to define the hypothetical surfaces A_2 and A_3. From Eq. 13.6,

$$(A_1 + A_2)F_{(1,2)(3,4)} = A_1\ F_{1(3,4)} + A_2\ F_{2(3,4)}$$

where $F_{(1,2)(3,4)}$ and $F_{2(3,4)}$ may be obtained from Fig. 13.6. Substituting for $A_1\ F_{1(3,4)}$ from Eq. 13.5 and combining expressions, find

$$A_1\ F_{1(3,4)} = A_1\ F_{13} + A_1\ F_{14}$$

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} - A_1\ F_{13} - A_2\ F_{2(3,4)}\right].$$

Substituting for $A_1\ F_{13}$ from Eq. 13.6, which may be expressed as

$$(A_1 + A_2)F_{(1,2)3} = A_1\ F_{13} + A_2\ F_{23}.$$

The desired relation is then

$$F_{14} = \frac{1}{A_1}\left[(A_1 + A_2)F_{(1,2)(3,4)} + A_2F_{23} - (A_1 + A_2)F_{(1,2)3} - A_2\ F_{2(3,4)}\right].$$ <

(b) For the prescribed dimensions and using Fig. 13.6, find these view factors:

Surfaces (1,2)(3,4) $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.45, \quad F_{(1,2)(3,4)} = 0.22$

Surfaces 23 $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{23} = 0.28$

Surfaces (1,2)3 $\quad (Y/X) = \frac{L_1 + L_2}{W} = 1, \quad (Z/X) = \frac{L_3}{W} = 1, \quad F_{(1,2)3} = 0.20$

Surfaces 2(3,4) $\quad (Y/X) = \frac{L_2}{W} = 0.5, \quad (Z/X) = \frac{L_3 + L_4}{W} = 1.5, \quad F_{2(3,4)} = 0.31$

Using the relation above, find

$$F_{14} = \frac{1}{(WL_1)}\left[(WL_1 + WL_2)0.22 + (WL_2)0.28 - (WL_1 + WL_2)0.20 - (WL_2)0.31\right]$$

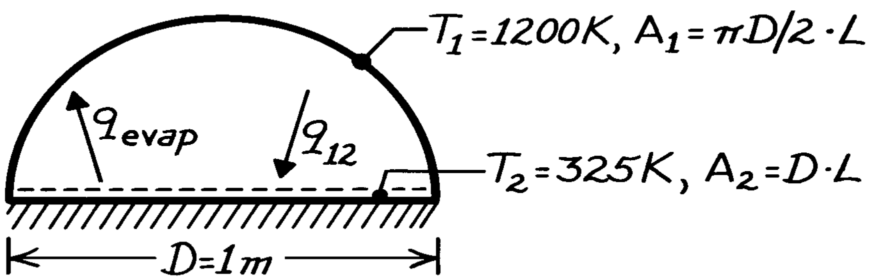
$$F_{14} = \left[2(0.22) + 1(0.28) - 2(0.20) - 1(0.31)\right] = 0.01.$$ <

PROBLEM 13.18

KNOWN: Surface temperature of a semi-circular drying oven.

FIND: Drying rate per unit length of oven.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for furnace wall and water, (2) Convection effects are negligible and bottom is insulated.

PROPERTIES: *Table A-6*, Water (325 K): $h_{fg} = 2.378\times10^6\,\mathrm{J/kg}$.

ANALYSIS: Applying a surface energy balance,

$$q_{12} = q_{evap} = \dot{m}\,h_{fg}$$

where it is assumed that the net radiation heat transfer to the water is balanced by the evaporative heat loss. From Eq. 13.13

$$q_{12} = A_1\,F_{12}\,\sigma\left(T_1^4 - T_2^4\right).$$

From inspection and the reciprocity relation, Eq. 13.3,

$$F_{12} = \frac{A_2}{A_1}F_{21} = \frac{D\cdot L}{(\pi D/2)\cdot L}\times1 = 0.637.$$

Hence

$$\dot{m}' = \frac{\dot{m}}{L} = \frac{\pi D}{2}F_{12}\,\sigma\frac{\left(T_1^4 - T_2^4\right)}{h_{fg}}$$

$$\dot{m}' = \frac{\pi\,(1\ \mathrm{m})}{2}\times0.637\times5.67\times10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2\cdot K^4}}\,\frac{(1200\ \mathrm{K})-(325\ \mathrm{K})^4}{2.378\times10^6\ \mathrm{J/kg}}$$

or

$$\dot{m}' = 0.0492\ \mathrm{kg/s\cdot m}. \qquad <$$

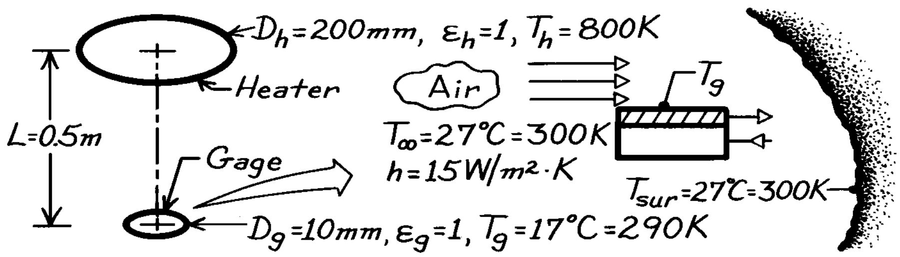
COMMENTS: Air flow through the oven is needed to remove the water vapor. The water surface temperature, T_2, is determined by a balance between radiation heat transfer to the water and the convection of latent and sensible energy from the water.

PROBLEM 13.34

KNOWN: Water-cooled heat flux gage exposed to radiant source, convection process and surroundings.

FIND: (a) Net radiation exchange between heater and gage, (b) Net transfer of radiation to the gauge per unit area of the gage, (c) Net heat transfer to the gage per unit area of gage, (d) Heat flux indicated by gage described in Problem 3.98.

SCHEMATIC:



ASSUMPTIONS: (1) Heater and gauge are parallel, coaxial discs having blackbody behavior, (2) $A_g << A_h$, (3) Surroundings are large compared to A_h and A_g.

ANALYSIS: (a) The net radiation exchange between the heater and the gage, both with blackbody behavior, is given by Eq. 13.13 having the form

$$q_{h-g} = A_h F_{hg}\,\sigma\left(T_h^4 - T_g^4\right) = A_g F_{gh}\,\sigma\left(T_h^4 - T_g\right).$$

Note the use of reciprocity, Eq. 13.3, for the view factors. From Eq. 13.8,

$$F_{gh} = D_h^2 / \left(4L^2 + D_h^2\right) = (0.2\text{m})^2 / \left(4\times 0.5^2\text{m}^2 + 0.2^2\text{m}^2\right) = 0.0385.$$

$$q_{h-g} = \left(\pi 0.01^2\text{m}^2/4\right)\times 0.0385\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left[800^4 - 290^4\right]\text{K}^4 = 69.0\ \text{mW}. \qquad <$$

(b) The net radiation *to* the gage per unit area will involve exchange with the heater and the surroundings. Using Eq. 13.14,

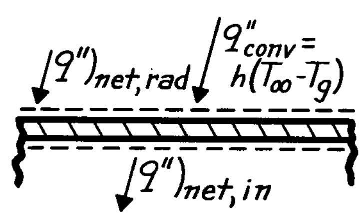
$$q''_{net,rad} = -q_g / A_g = q_{h-g}/A_g + q_{sur-g}/A_g.$$

The net exchange with the surroundings is

$$q_{sur-g} = A_{sur}F_{sur-g}\,\sigma\left(T_{sur}^4 - T_g 4\right) = A_g F_{g-sur}\,\sigma\left(T_{sur}^4 - T_g^4\right).$$

$$q''_{net,rad} = \frac{69.0\times 10^{-3}\text{W}}{\pi(0.01\text{m})^2/4} + (1-0.0385)5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(300^4 - 290^4\right)\text{K}^4 = 934.5\,\text{W/m}^2. \qquad <$$

(c) The net heat transfer rate to the gage per unit area of the gage follows from the surface energy balance



$$q''_{net,in} = q''\big)_{net,rad} + q''_{conv}$$

$$q''_{net,in} = 934.5\,\text{W/m}^2 + 15\,\text{W/m}^2\cdot\text{K}\,(300-290)\,\text{K}$$

$$q''_{net,in} = 1085\,\text{W/m}^2. \qquad <$$

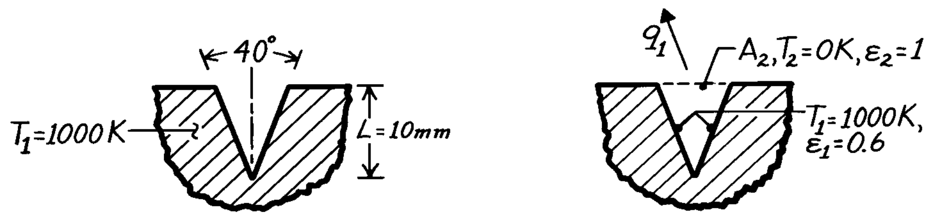
(d) The heat flux gage described in Problem 3.98 would experience a net heat flux to the surface of 1085 W/m². The irradiation to the gage from the heater is $G_g = q_{h\to g}/A_g = F_{gh}\,\sigma T_h^4 = 894\ \text{W/m}^2$. Since the gage responds to net heat flux, there would be a systematic error in sensing irradiation from the heater.

PROBLEM 13.43

KNOWN: Long V-groove machined in an isothermal block.

FIND: Radiant flux leaving the groove to the surroundings and effective emissivity.

SCHEMATIC:



ASSUMPTIONS: (1) Groove surface is diffuse-gray, (2) Groove is infinitely long, (3) Block is isothermal.

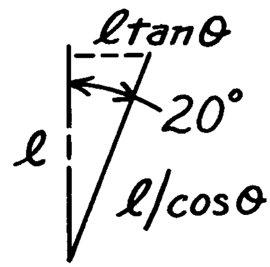
ANALYSIS: Define the hypothetical surface A_2 with $T_2 = 0$ K. The net radiation leaving A_1, q_1, will pass to the surroundings. From the two surface enclosure analysis, Eq. 13.23,

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Recognize that $\varepsilon_2 = 1$ and that from reciprocity, $A_1\,F_{12} = A_2\,F_{21}$ where $F_{21} = 1$. Hence,

$$\frac{q_1}{A_2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1}\dfrac{A_2}{A_1} + 1}$$

With $A_2/A_1 = 2\ell\ \tan 20° / \left(2\ell / \cos 20°\right) = \sin 20°$, find



$$q_1'' = \frac{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000^4 - 0\right)\text{K}^4}{\dfrac{(1-0.6)}{0.6}\times\sin 20° + 1} = 46.17\ \text{kW/m}^2. \quad <$$

The effective emissivity of the groove follows from the definition given in Problem 13.42 as the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the cavity opening and at the same temperature as the cavity surface. For the present situation,

$$\varepsilon_e = \frac{q_1''}{E_b\left(T_1\right)} = \frac{q_1''}{\sigma T_1^4} = \frac{46.17\times10^{+3}\,\text{W/m}^2}{5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1000\ \text{K}\right)^4} = 0.814. \quad <$$

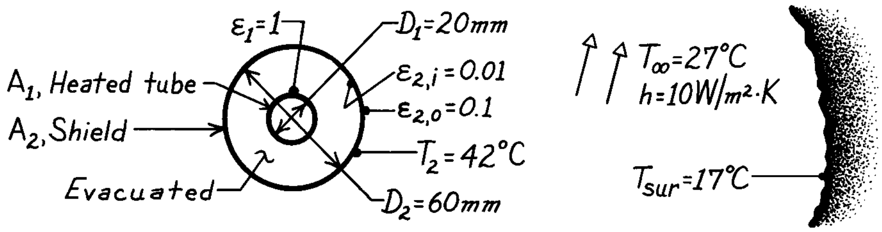
COMMENTS: Note the use of the hypothetical surface defined as black at 0 K. This surface does not emit and absorbs all radiation on it; hence, is the radiant power to the surroundings.

PROBLEM 13.62

KNOWN: Heated tube with radiation shield whose exterior surface is exposed to convection and radiation processes.

FIND: Operating temperature for the tube under the prescribed conditions.

SCHEMATIC:

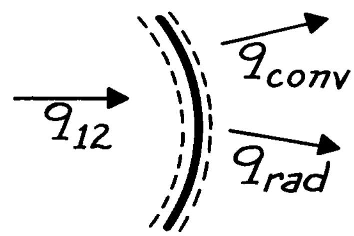


ASSUMPTIONS: (1) Steady-state conditions, (2) No convection in space between tube and shield, (3) Surroundings are large compared to the shield and are isothermal, (4) Tube and shield are infinitely long, (5) Surfaces are diffuse-gray, (6) Shield is isothermal.

ANALYSIS: Perform an energy balance on the shield.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_{12} - q_{conv} - q_{rad} = 0$$



where q_{12} is the net radiation exchange between the tube and inner surface of the shield, which from Eq. 13.25 is,

$$-q_{12} = \frac{A_1 \sigma \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}} \frac{D_1}{D_2}}$$

Using appropriate rate equations for q_{conv} and q_{rad}, the energy balance is

$$\frac{A_1 \sigma \left(T_1^4 - T_2^4\right)}{1 + \frac{1-\varepsilon_{2,i}}{\varepsilon_{2,i}} \frac{D_1}{D_2}} - hA_2 \left(T_2 - T_\infty\right) - \varepsilon_{2,o} A_2 \sigma \left(T_2^4 - T_{sur}^4\right) = 0$$

where $\varepsilon_1 = 1$. Substituting numerical values, with $A_1/A_2 = D_1/D_2$, and solving for T_1,

$$\frac{(20/60) \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(T_1^4 - 315^4\right) K^4}{1 + (1 - 0.01/0.01)(20/60)} - 10\ W/m^2 \cdot K (315 - 300) K$$

$$-0.1 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(315^4 - 290^4\right) K^4 = 0$$

$$T_1 = 745\ K = 472°C. \qquad <$$

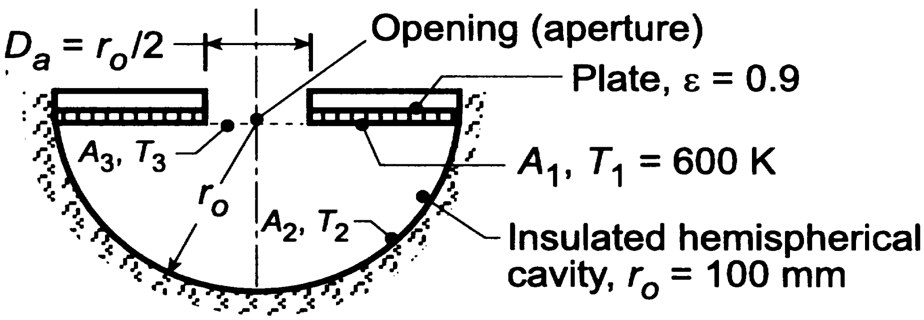
COMMENTS: Note that all temperatures are expressed in kelvins. This is a necessary practice when dealing with radiation and convection modes.

PROBLEM 13.69

KNOWN: Blackbody simulator design consisting of a heated circular plate with an opening over a well insulated hemispherical cavity.

FIND: (a) Radiant power leaving the opening (aperture), $D_a = r_o/2$, (b) Effective emissivity of the cavity, ε_e, defined as the ratio of the radiant power leaving the cavity to the rate at which the circular plate would emit radiation if it were black, (c) Temperature of hemispherical surface, T_{hc}, and (d) Compute and plot ε_e and T_{hc} as a function of the opening aperture in the circular plate, D_a, for the range $r_o/8 \le D_a \le r_o/2$, for plate emissivities of $\varepsilon_p = 0.5$, 0.7 and 0.9.

SCHEMATIC:



ASSUMPTIONS: (1) Plate and hemispherical surface are diffuse-gray, (2) Uniform radiosity over these same surfaces.

ANALYSIS: (a) The simulator can be treated as a three-surface enclosure with one re-radiating surface (A_2) and the opening (A_3) as totally absorbing with no emission into the cavity (T_3 = 300 K). The radiation leaving the cavity is the net radiation leaving A_1, q_1 which is equal to $-q_3$. Using Eq. 13.30,

$$q_{cav} = q_1 = -q_3 = \frac{\sigma\left(T_1^4 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + \left[A_1F_{13} + \left[(1/A_1F_{12}) + (1/A_3F_{32})\right]^{-1}\right]^{-1} + (1-\varepsilon_3)/\varepsilon_3 A_3} \qquad (1)$$

Using the summation rule and reciprocity, evaluate the required view factors:

$$F_{11} + F_{12} + F_{13} = 1 \qquad F_{13} = 0 \qquad F_{12} = 1$$

$$F_{31} + F_{32} + F_{33} = 1 \qquad F_{32} = 1.$$

Substituting numerical values with $\varepsilon_3 = 1$, T_3 = 300 K, $A_1 = \pi\left(r_o^2 - (r_o/4)^2\right) = 15\pi r_o^2/16 = 2.945 \times 10^{-2}\ m^2$, $A_3 = \pi r_a^2 = \pi\left(r_o/4\right)^2 = 1.963 \times 10^{-3}\ m^2$ and $A_1/A_3 = 15$, and multiplying numerator and denominator by A_1,

$$q_{cav} = q_1 = \frac{A_1\sigma\left(T_1 - T_3^4\right)}{(1-\varepsilon_1)/\varepsilon_1 + \left\{F_{13} + \left[(1/F_{12}) + (A_1/A_3F_{32})\right]^{-1}\right\}^{-1} + 0} \qquad (2)$$

Continued …..

PROBLEM 13.69 (Cont.)

$$q_{cav} = q_1 = \frac{2.945\times10^{-2}\,\mathrm{m}^2 \times 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(600^4 - 300^4\right)\mathrm{K}^4}{(1-0.9)/0.9 + \left\{0 + \left[1+(15/1)\right]^{-1}\right\}^{-1} + 0} = 12.6\,\mathrm{W}$$ <

(b) The effective emissivity is the ratio of the radiant power leaving the cavity to that from a blackbody having the area of the opening and temperature of the inner surface of the cavity. That is,

$$\varepsilon_e = \frac{q_{cav}}{A_3 \sigma T_1^4} = \frac{12.6\,\mathrm{W}}{1.963\times10^{-3}\,\mathrm{m}^2 \times 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4} \times (600\,\mathrm{K})^4} = 0.873 \qquad (3)$$ <

(c) From a radiation balance on A_1, find J_1,

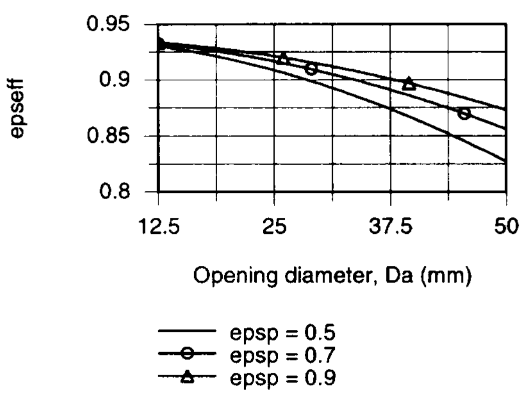
$$q_1 = 12.6\,\mathrm{W} = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{\sigma 600^4 - J_1}{(1-09)/0.9A_1} \qquad J_1 = 7301\,\mathrm{W/m^2} \qquad (4)$$

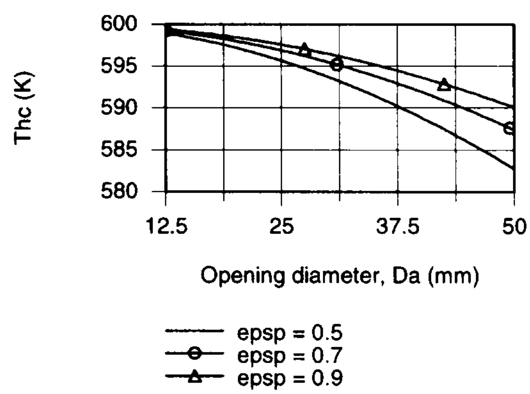
From a radiation balance on A_2 with $J_3 = E_{b3} = \sigma T_3^4 = 459.9\ \mathrm{W/m^2}$ and $J_2 = \sigma T_2^4$, find

$$\frac{J_2 - J_1}{(1/A_1F_{12})} + \frac{J_2 - J_3}{(1/A_3F_{32})} = \frac{J_2 - 7301\,\mathrm{W/m^2}}{\left(1/2.945\times10^{-2}\,\mathrm{m}^2\right)} + \frac{J_2 - 459.9}{\left(1/1.963\times10^{-3}\,\mathrm{m}^2\right)} = 0 \qquad (5)$$

$$J_2 = 6873\,\mathrm{W/m^2} \qquad T_2 = 590\,\mathrm{K.}$$ <

(d) Using the foregoing equations in the *IHT* workspace, ε_e and T_2 were computed and plotted as a function of the opening, D_a, for selected plate emissivities, ε_p.





From the upper-left graph, ε_e decreases with increasing opening, D_a, as expected. In the limit as $D_a \rightarrow 0$, $\varepsilon_3 \rightarrow 1$ since the cavity becomes a complete enclosure. From the upper-right graph, T_{hc}, the temperature of the re-radiating hemispherical surface decreases as D_a increases. In the limit as $D_a \rightarrow 0$, T_2 will approach the plate temperature, T_p = 600 K. The effect of decreasing the plate emissivity is to decrease ε_e and decrease T_2. Why is this so?

Continued …..

PROBLEM 13.69 (Cont.)

COMMENTS: The *IHT Radiation, Tool, Radiation Tool, Radiation Exchange Analysis, Three-Surface Enclosure with Re-radiating Surface*, is especially convenient to perform the parametric analysis of part (c). A copy of the *IHT* workspace that can generate the above graphs is shown below.

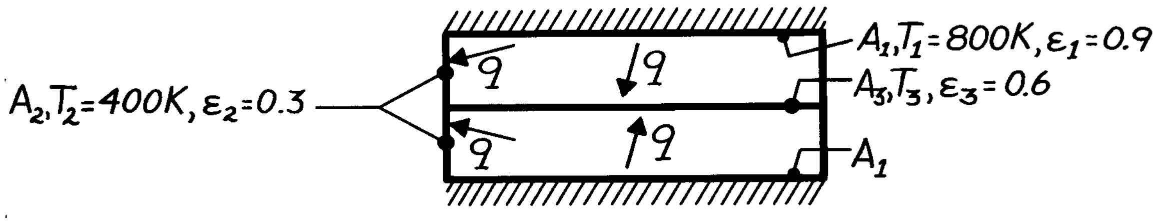
```
// Radiation Tool – Radiation Exchange Analyses, Reradiating Surface
/* For the three-surface enclosure A1, A3 and the reradiating surface A2, the net rate of radiation transfer
from the surface A1 to surface A3 is */
q1 = (Eb1 – Eb3) / ( (1 – eps1)/(eps1 * A1) + 1/(A1 * F13 + 1/(1/(A1 * F12) + 1/(A3 * F32))) + (1 –
eps3)/(eps3 * A3))  //  Eq 13.30
/* The net rate of radiation transfer from surface A3 to surface A1 is */
q3 = q1
/* From a radiation energy balance on A2, */
(J2 – J1) / (1/(A2 * F21)) + (J2 – J3)/(1/(A2 * F23) ) = 0          // Eq 13.31
/* where the radiosities J1 and J3 are determined from the radiation rate equations expressed in terms of
the surface resistances, Eq 13.22 */
q1 = (Eb1 – J1) / ((1 – eps1) / (eps1 * A1))
q3 = (Eb3 – J3) / ((1 – eps3) / (eps3 * A3))
// The blackbody emissive powers for A1 and A3 are
Eb1 = sigma * T1^4
Eb3 = sigma * T3^4
// For the reradiating surface,
J2 = Eb2
Eb2 = sigma * T2^4                // Stefan-Boltzmann constant, W/m^2·K^4
sigma = 5.67E-8

// Effective emissivity:
epseff = q1 / (A3 * Eb1)                    // Eq (3)

// Areas:
A1 = pi * ( ro^2 · ra^2)
A2 = 0.5 * pi * (2 * ro)^2                  // Hemisphere, As = 0.5 * pi * D^2
A3 = pi * ra^2

// Assigned Variables
T1 = 600                          // Plate temperature, K
eps1 = 0.9                        // Plate emissivity
T3 = 300                          // Opening temperature, K; Tsur
eps3 = 0.9999                     // Opening emissivity; not zero to avoid divide-by-zero error
ro = 0.1                          // Hemisphere radius, m
Da = 0.05                         // Opening diameter; range ro/8 to ro/2; 0.0125 to 0.050
Da_mm = Da * 1000                           // Scaling for plot
Ra = Da / 2                       // Opening radius
```

PROBLEM 13.77

KNOWN: Dimensions, surface radiative properties, and operating conditions of an electrical furnace.

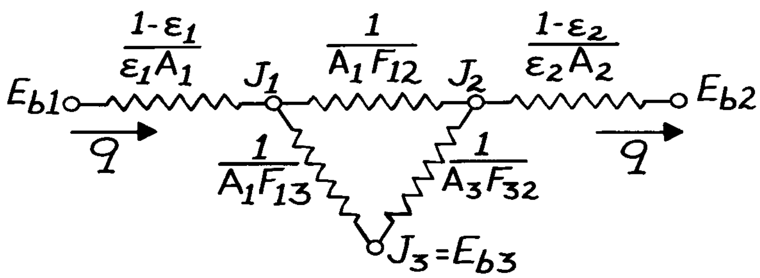
FIND: (a) Equivalent radiation circuit, (b) Furnace power requirement and temperature of a heated plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Opaque, diffuse-gray surfaces, (3) Negligible plate temperature gradients, (4) Back surfaces of heater are adiabatic, (5) Convection effects are negligible.

ANALYSIS: (a) Since there is symmetry about the plate, only one-half (top or bottom) of the system need be considered. Moreover, the plate must be adiabatic, thereby playing the role of a re-radiating surface.



<

(b) Note that $A_1 = A_3 = 4\ m^2$ and $A_2 = (0.5\ m \times 2\ m)4 = 4\ m^2$. From Fig. 13.4, with X/L = Y/L = 4, $F_{13} = 0.62$. Hence

$$F_{12} = 1 - F_{13} = 0.38, \quad \text{and} \quad F_{32} = F_{12} = 0.38.$$

It follows that

$$A_1F_{12} = 4(0.38) = 1.52\,m^2$$

$$A_1F_{13} = 4(0.62) = 2.48\,m^2, \qquad (1-\varepsilon_1)/\varepsilon_1 A_1 = 0.1/3.6\,m^2 = 0.0278\,m^{-2}$$

$$A_3F_{32} = 4(0.38) = 1.52\,m^2, \qquad (1-\varepsilon_2)/\varepsilon_2 A_2 = 0.7/1.2\,m^2 = 0.583\,m^{-2}.$$

Also,

$$E_{b1} = \sigma T_1^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(800\ K)^4 = 23{,}224\,W/m^2,$$

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\,W/m^2\cdot K^4\,(400\ K)^4 = 1452\,W/m^2.$$

The system forms a three-surface enclosure, with one surface re-radiating. Hence the net radiation transfer from a single heater is, from Eq. 13.30,

$$q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[1/A_1F_{13} + 1/A_3F_{32}\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Continued …..

PROBLEM 13.77 (Cont.)

$$q_1 = \frac{(23,224-1452)\,\mathrm{W/m^2}}{(0.0278+0.4061+0.583)\,\mathrm{m^{-2}}} = 21.4\,\mathrm{kW}.$$

The furnace power requirement is therefore $q_{elec} = 2q_1 = 43.8$ kW, with <

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1}.$$

where

$$J_1 = E_{b1} - q_1 \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 23,224\,\mathrm{W/m^2} - 21,400\,\mathrm{W} \times 0.0278\,\mathrm{m^{-2}}$$

$$J_1 = 22,679\,\mathrm{W/m^2}.$$

Also,

$$J_2 = E_{b2} - q_2 \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 1,452\,\mathrm{W/m^2} - (-21,400\,\mathrm{W}) \times 0.583\,\mathrm{m^{-2}}$$

$$J_2 = 13,928\,\mathrm{W/m^2}.$$

From Eq. 13.31,

$$\frac{J_1 - J_3}{1/A_1F_{13}} = \frac{J_3 - J_2}{1/A_3F_{32}}$$

$$\frac{J_1 - J_3}{J_3 - J_2} = \frac{A_3F_{32}}{A_1F_{13}} = \frac{1.52}{2.48} = 0.613$$

$$1.613J_3 = J_1 + 0.613J_2 = 22,629 + 8537 = 31,166\,\mathrm{W/m^2}$$

$$J_3 = 19,321\,\mathrm{W/m^2}$$

Since $J_3 = E_{b3}$,

$$T_3 = (E_{b3}/\sigma)^{1/4} = \left(19,321/5.67\times10^{-8}\right)^{1/4} = 764\ \mathrm{K}.$$ <

COMMENTS: (1) To reduce q_{elec}, the sidewall temperature T_2, should be increased by insulating it from the surroundings. (2) The problem must be solved by simultaneously determining J_1, J_2 and J_3 from the radiation balances of the form

$$\frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = A_1F_{12}(J_1 - J_2) + A_1F_{13}(J_1 - J_3)$$

$$\frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = A_2F_{21}(J_2 - J_1) + A_2F_{23}(J_2 - J_3)$$

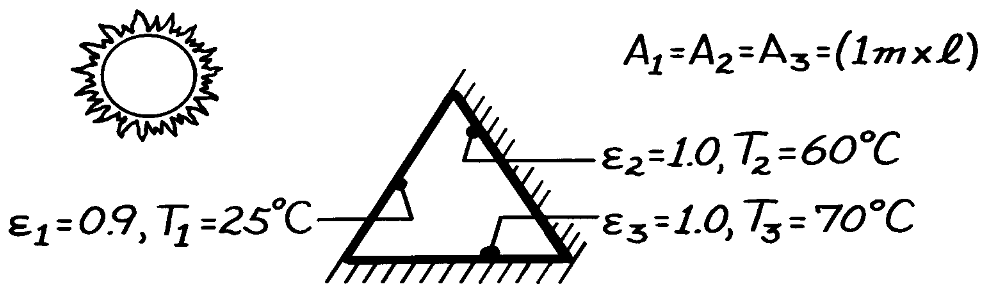
$$0 = A_1F_{13}(J_3 - J_1) + A_2F_{23}(J_3 - J_2).$$

PROBLEM 13.78

KNOWN: Geometry and surface temperatures and emissivities of a solar collector.

FIND: Net rate of radiation transfer to cover plate due to exchange with the absorber plates.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal surfaces with uniform radiosity, (2) Absorber plates behave as blackbodies, (3) Cover plate is diffuse-gray and opaque to thermal radiation exchange with absorber plates, (4) Duct end effects are negligible.

ANALYSIS: Applying Eq. 13.21 to the cover plate, it follows that

$$E_{b1} - J_1 = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} \sum_{j=1}^{N} \frac{J_1 - J_j}{\left(A_i F_{ij}\right)^{-1}} = \frac{1-\varepsilon_1}{\varepsilon_1 A_1}\left[A_1 F_{12}\left(J_1 - J_2\right) + A_1 F_{13}\left(J_1 - J_3\right)\right].$$

From symmetry, $F_{12} = F_{13} = 0.5$. Also, $J_2 = E_{b2}$ and $J_3 = E_{b3}$. Hence

$$E_{b1} - J_1 = 0.0556\left(2J_1 - E_{b2} - E_{b3}\right)$$

or with $E_b = \sigma T^4$,

$$1.111 J_1 = E_{b1} + 0.0556\left(E_{b2} + E_{b3}\right)$$

$$1.111 J_1 = 5.67\times10^{-8}\,(298)^4\ \mathrm{W/m^2} + 0.0556\left(5.67\times10^{-8}\right)\left[(333)^4 + (343)^4\right]\mathrm{W/m^2}$$

$$J_1 = 476.64\ \mathrm{W/m^2}$$

From Eq. 13.19 the net rate of radiation transfer *from* the cover plate is then

$$q_1 = \frac{E_{b1} - J_1}{\left(1-\varepsilon_1\right)/\varepsilon_1 A_1} = \frac{5.67\times10^{-8}\,(298)^4 - 476.64}{\left(1-0.9\right)/0.9(\ell)} = \left(-265.5\ell\right)\mathrm{W}.$$

The net rate of radiation transfer *to* the cover plate per unit length is then

$$q_1' = \left(q_1/\ell\right) = 266\ \mathrm{W/m}. \qquad <$$

COMMENTS: Solar radiation effects are not relevant to the foregoing problem. All such radiation transmitted by the cover plate is completely absorbed by the absorber plate.