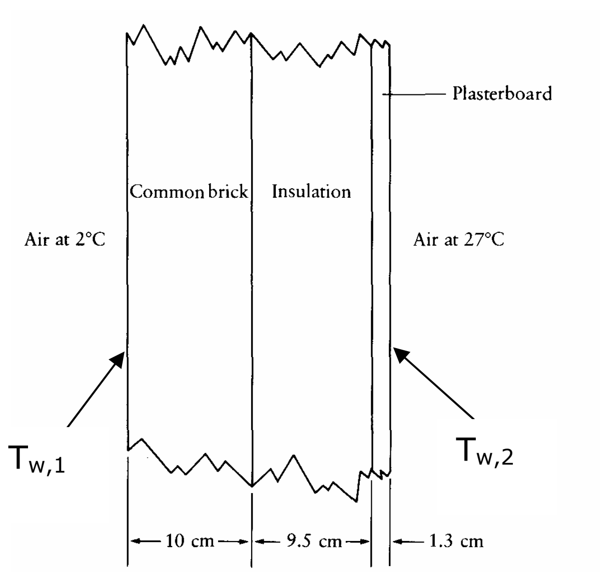
Int. To Heat Transfer (ME 411) Final Exam July 29, 2005
Name: (1 Hour)

Solve *only* two problems. If solutions involve an iterative process multiple iterations are not necessary.

Problem 1 (Conduction): An aluminum saucepan has a handle that is riveted to its wall. The handle itself is made of cast aluminum (k=164 w/mK) and is to have attached a plastic grip that is comfortable to grasp. Before selecting a plastic, it is necessary to have information on the temperature of the aluminum handle. The aluminum handle can be considered as a rod 11 mm in diameter and 45 mm long. When being used over a stove burner, the ambient temperature is 44°C, and the temperature at the base of the handle reaches 110°C. For a convection heat transfer coefficient (h) of 8 W/m^2K, determine

(1) The temperature profile if the tip is insulated. (5 points)
(2) The temperature profile if the tip is *not* insulated. (5 points)
(3) **(Bonus)** The heat transferred by the handle for both cases. (3 points)

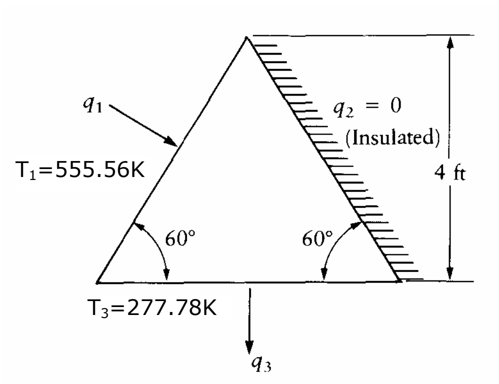
Problem 2 (Free Convection/Conduction): A vertical wall that is shown here. The outside brick is 10 cm thick, and the inside panel is 1.3 cm-thick plaster board. The brick and plasterboard are separated by 9.5 cm of glass-fiber insulation. On the brick side is air at 2°C, while on the plasterboard side is air at 27°C. The wall is 2.5 m tall. How much heat is transferred through wall per unit width? (15 points)
(Hint: Assume $T_{w,1}$ and $T_{w,2}$ as 10°C and 20°C, respectively.)



-Properties of air at
T=275K: ρ=1.295 kg/m^3, k=0.02426 w/mK, C_p=1005.5 j/kg/K, α=0.17661e-4 m^2/s, Pr=0.713, ν = 12.59e-6 m^2/s
T=300K: ρ=1.177 kg/m^3, k=0.02624 w/mK, C_p=1005.7 j/kg/K, α=0.2216e-4 m^2/s, Pr=0.708, ν = 16.68e-6 m^2/s
-Thermal conductivity of the brick, glass-fiber and plaster: 0.45, 0.035, and 0.814 w/mK, respectively.

Problem 3 (Radiation): A very long broiler whose proposed design is to be evaluated is shown here. The oven cross section is an equilateral triangle with one side insulated. The heater surface is maintained at 555.56K while the bottom is at 277.78K. Find

(1) The view factor for each surface in all directions (F11, F12, F13, F22, …) (4 points)
(2) The temperature of the insulated surface. (4 points)
(3) The heat that must be supplied to each of the isothermal surfaces. (2 points)



PROBLEM 1.
Pin Fin Problem.

Steady state Conduction

$D = 11\,mm = 0.011\,m$, $L = 45\,mm = 0.045\,m$

$T_\infty = 44\,°C$, $T_W = 110\,°C$, $\bar{h}_c = 8\,W/m^2K$, $K = 164\,W/mK$

$m = \sqrt{\dfrac{\bar{h}_c P}{K A_c}}$, $P = \pi D = \pi(0.011) = 0.03456\,m$
$A = \pi D^2/4 = \pi(0.011)^2/4 = 9.5\times10^{-5}\,m^2$

$$\rightarrow m = \sqrt{\frac{8\times0.03456}{164\times9.5\times10^{-5}}} = 4.212$$

(1) insulated tip (Table 3.4, Case B)

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL} = \frac{\cosh[4.212\times(0.045-x)]}{\cosh(4.212\times0.045)}$$

$$\rightarrow \frac{T-T_\infty}{T_b-T_\infty} = \frac{\cosh(0.1896-4.212x)}{1.018}$$

or $\rightarrow \dfrac{T-44}{110-44} = \dfrac{\cosh(0.19-4.21x)}{1.02}$ or $T = 44 + 64.83\cosh(0.19-4.21x)$ Ans.

(2) Uninsulated tip (Table 3.4, Case A)

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + \frac{h}{mK}\sinh(L-x)}{\cosh mL + h/mK \sinh mL}$$

or
$$\frac{T-44}{110-44} = \frac{\cosh(0.19-4.21x) + \frac{8}{(4.212)(164)}\sinh(0.19-4.21x)}{\cosh(4.212\times0.045) + \frac{8}{4.212\times164}\sinh(4.212\times0.045)}$$

or
$$T = 44 + 64.69\left[\cosh(0.19-4.21x) + 0.01158\sinh(0.19-4.21x)\right]$$
Ans.

Problem 1 (cont'd)

(3) using Table 3.4 for cases "B" and "A" you can find q_p:

$$M = \sqrt{h_c P K A_c}\,\theta_b = \sqrt{8(0.03456)(9.5\times10^{-5})(164)}\,[110.441]$$

$$\rightarrow M = 4.33$$

→ Insulated $q_{f,i} = M \tanh mL = 4.33 \times \tanh(4.212 \times 0.045)$

$$\rightarrow q_{f,i} = 0.813 \text{ W}$$

→ Uninsulated

$$q_{f,u} = M \frac{\sinh mL + \frac{h}{mK}\cosh mL}{\cosh mL + \frac{h}{mK}\sinh mL}$$

$$= 4.33 \frac{\sinh(0.19) + \frac{8}{4.212\times164}\cosh(0.19)}{\cosh(0.19) + \frac{8}{4.212\times164}\sinh(0.19)}$$

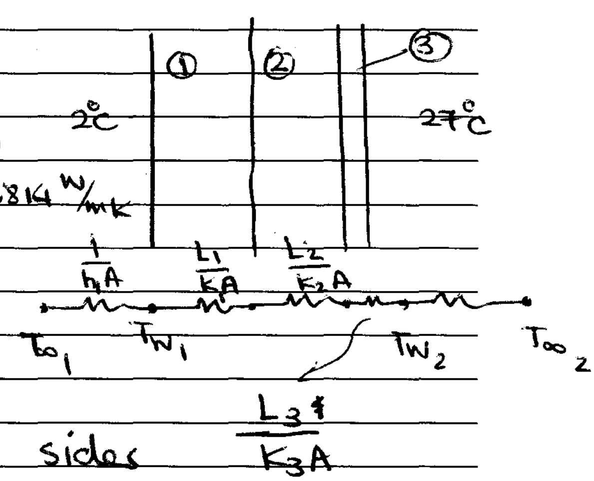
$$\rightarrow q_{f,u} = 0.86 \text{ W}$$

Problem 2.

The circuit is shown here,

$L_1 = 0.1$ m, $L_2 = 0.095$ m, $L_3 = 0.013$ m

$K_1 = 0.45$ W/mK, $K_2 = 0.035$ W/mK, $K_3 = 0.814$ W/mK



$U_{\infty_1} = U_{\infty_2} = 0 \rightarrow$ Free Convection

we need to find Ra_L for both sides

but we need to know T_{w_1} and T_{w_2}, using the suggested $T_{w_1} = 10°C$, $T_{w_2} = 20°C$ we have,

$$Ra_{L,1} = \frac{g\beta(T_{W1}-T_{\infty 1})}{\nu\alpha} L^3 = \frac{9.81\overbrace{\left(\frac{1}{275}\right)}^{\beta}(10-2)(2.5)^3}{(12.59\times10^{-6})(0.17661\times10^{-4})}$$

$$\rightarrow Ra_{L,1} = 2.02\times10^{10} > 10^9 \quad \text{Turbulent}$$

$$Ra_{L,2} = \frac{g\beta(-T_{W2}+T_{\infty 2})L^3}{\nu\alpha} = \frac{9.81\left(\frac{1}{300}\right)(-20+27)(2.5)^3}{(15.68\times10^{-6})(0.2216\times10^{-4})}$$

$$\rightarrow Ra_{L,2} = 1.044\times10^{10} > 10^9 \quad \text{Turbulent}$$

to find the convection heat transfer coefficient we need to use Eq. (8.26)

$$\overline{Nu_L} = \left[8.25 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2$$

for Ra_{L_1} we get $Nu_{L_1} = 316.26$ and $\overline{h}_1 = 3.069\ W/m^2K$

and for Ra_{L_2} we get $\overline{Nu}_{L_2} = 255.24$, $\overline{h}_2 = 2.679\ W/m^2K$

Since we've gussed for T_{w_1} and T_{w_2} we need to correct these. from Conduction for a composite wall we have

$$q'' = \frac{\Delta T}{RA} = \frac{T_{\infty 1} - T_{\infty 2}}{\sum RA} = \frac{2-27}{\frac{1}{3.069}+\frac{0.1}{0.45}+\frac{0.095}{0.035}+\frac{0.013}{0.814}+\frac{1}{2.67}}$$

Then $q'' = -6.85\ W/m^2$

now q'' is constant throughout the circuit it means

$$q'' = \frac{T_{\infty 1} - T_{W1}}{\frac{1}{h_1}} \Rightarrow -6.85 = \frac{2 - T_{W1}}{\frac{1}{3.069}} \rightarrow T_{W1} = 4.23^\circ C$$

and

$$q'' = \frac{T_{W2} - T_{\infty 2}}{\frac{1}{h_2}} \longrightarrow -6.85 = \frac{T_{W2} - 27}{\frac{1}{2.679}} \rightarrow T_{W2} = 24.4^\circ c$$

Now you can ~~use~~ the corrected T_{W1} and T_{W2} values and find the properties. You will get

for $T_{W1} = 4.23\ ^\circ c$, $T_{W2} = 24.4^\circ c$

$Ra_{L_1} = 5.5 \times 10^9$, $\quad Ra_{L_2} = 3.88 \times 10^9$ Still Turbulent

$\bar{h}_1 = 2.04\ W/m^2K$, $\quad \bar{h}_2 = 1.96\ W/m^2K$

new $q'' = \dfrac{-27.42}{\frac{1}{2.04} + 0.222 + 2.71 + 0.016 + \frac{1}{1.96}} = -6.33\ W/m^2$

Then $T_{W1} = \dfrac{6.33}{2.04} + 2 = 5.1^\circ c$

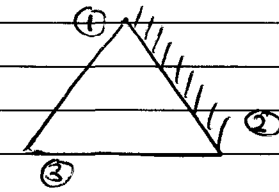
and $T_{W2} = 27 - \dfrac{6.33}{1.96} = 23.8\ ^\circ c$

you can repeat more until converges to the final solution, Finally you will get

$T_{W1} = 4.8^\circ C$, $\quad T_{W2} = 23.9\ ^\circ C$, $\quad q'' = 6.46\ W/m^2$

The $q' = q'' \times L = 6.46 \times 2.5 = 16.2\ W/m$

Problem 3:



Sumation rule for ①

$F_{11}+F_{12}+F_{13}=1$
and also
$F_{11}=0$

$\left.\begin{array}{l} F_{11}+F_{12}+F_{13}=1 \\ F_{11}=0 \end{array}\right\} F_{12}+F_{13}=1$

for surface ②

$\left.\begin{array}{l} F_{22}+F_{21}+F_{23}=1 \\ F_{22}=0 \end{array}\right\} F_{21}+F_{23}=1$

for surface ③

$\left.\begin{array}{l} F_{33}+F_{31}+F_{32}=1 \\ F_{33}=0 \end{array}\right\} \longrightarrow F_{31}+F_{32}=1$

The surface areas are the same, $A_1=A_2=A_3$ then

$F_{12}\,A_1=F_{21}\,A_2 \longrightarrow F_{12}=F_{21}$

$F_{13}\,A_1=F_{31}\,A_3 \longrightarrow F_{13}=F_{31}$

$F_{23}\,A_2=F_{32}\,A_3 \longrightarrow F_{23}=F_{32}$

Comaring with these, equations we get

$$F_{12}=F_{21}=F_{13}=F_{31}=F_{23}=F_{32}=1/2$$

and $F_{11}=F_{22}=F_{33}=0$

Now using energy balane for surface ② we have
[or use Eq. (13.14)]

$$q_2=q_{21}+q_{23}=F_{21}A_2(T_2^4-T_1^4)+F_{23}A_2(T_2^4-T_3^4)$$

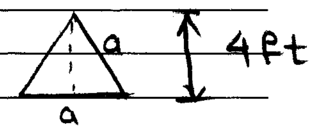
and $q_2=0$ (insulated).

Problem 3 (cont'd)
we get

$$0 = 0.5 \times A_1 (T_2^4 - 555.56^4) + 0.5 \times A_2 (T_2^4 - 277.78^4)$$

(1) or $T_2 = 474.303$ K Ans.

(2) for the triangle we have



$a^2 = (\frac{a}{2})^2 + 4^2 \rightarrow a = 4.62$ ft
or $a = 0.16$ m

$$q_1 = q_{12} + q_{13}$$

$$= F_{12} A_1 (T_1^4 - T_2^4) + F_{13} A_1 (T_1^4 - T_3^4)$$

Then $q_1' = 0.611$ KW/m Ans.

and

Energy balance for Side 3

$$q_3' = -q_1' = -.611 \text{ KW/m}$$ Ans.