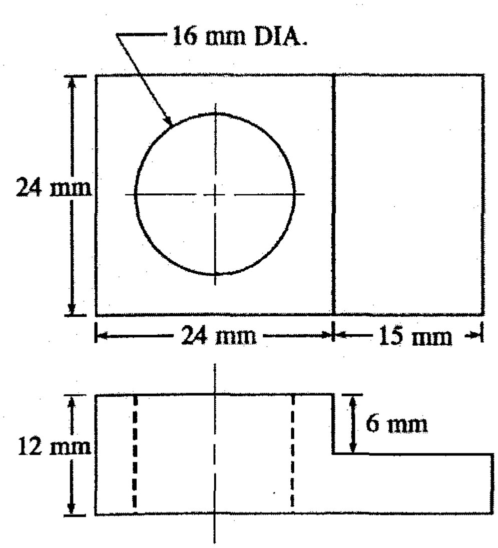
Int. To Heat Transfer (ME 411) Quiz 2 July 11, 2005
Name: (1 Hour)

Problem 1 (10 points): A rod of length L has one end maintained at temperature T_0 and is exposed to an environment at temperature T_∞. An electrical heating element is placed in the rod so that heat is generated uniformly along the length at a rate $\dot{q}$. Derive an expression

(a) For the temperature distribution in the rod, (3 points)
(b) For the total heat transferred to the environment, (3 points)
(c) Obtain an expression for the value of $\dot{q}$ that will make the heat transfer zero at the end which is maintained at T_0. (4 points)
(d) Bonus If $\dot{q} = a + b.x$, find the expression for the temperature. (5 points)

Problem 2 (10 points): The part shown below is machined from Stainless Steel (ρ=7978 kg/m^3, k=18.9 w/mk, C_p=559 j/kg/K). When the part is removed from the heat treating furnace, it has an initial uniform temperature of 600°C. All surfaces of the part are exposed to a flow of coolant with a temperature of 30°C and a heat transfer coefficient of h=25 W/m^2K.

(a) Check the validity of the lumped capacitance method for this problem. (3 points)
(b) Calculate how long it will take for the part to cool to 54°C. (4 points)
(c) When the part has a temperature of 54°C, what are the instantaneous rate of heat loss from the part and the instantaneous rate of cooling (in °C/s). (3 points)

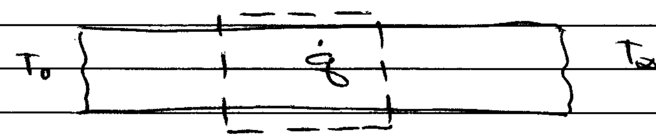


2005 – Summer II

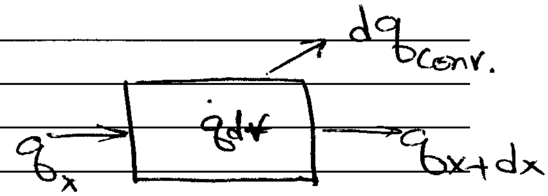
Quiz 2

Problem 1:

Similar to the example in the class, we have



Energy Balance for



$E_{in} - E_{out} = \overset{0\ (S.S.)}{\cancel{E_{st}}} - E_{gen}$ (1)

$$q_x - q_{x+dx} = -\dot{q}\, d\forall$$

$$q_x = -KA\frac{\partial T}{\partial x} \rightarrow \frac{dq_x}{dx} = -KA\frac{d^2T}{dx^2} \quad \text{for } K = \text{const.}$$

$$q_{conv.} = hA(T - T_\infty) \rightarrow dq_{conv.} = h\,dA\,(T - T_\infty)$$

pluging into Eq. (1) and simplify

$\rightarrow$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) + \frac{\dot{q}}{K} = 0$$

or if $\theta = T - T_\infty$

$$\theta'' - m^2\theta + \dot{q}/K = 0 \quad , \quad m^2 = \frac{hP}{KA}$$

The solution to this differential equation (which is non-homogeneous but linear) is given as

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + \frac{\dot{q}}{m^2K}$$

Prob. 1 / Cont'd

The boundary conditions are

$$\theta(0) = \theta_0 \quad (a) \quad , \quad -K\cancel{A}\frac{d\theta}{dx}\Big|_{x=L} = h\cancel{A}\theta\Big|_{x=L} = h\cancel{A}\theta_L \quad (b)$$

using (a)

$$\theta_0 = C_1 + C_2 + \frac{\dot{q}}{m^2 K}$$

using (b) $$-C_1 e^{mL} + C_2 e^{-mL} = \frac{h\theta_L}{Km}$$

multiply by e^{mL} $\longrightarrow -C_1 e^{2mL} + C_2 = \frac{h\theta}{Km} e^{mL}$

Subtract from (a) $\longrightarrow$ $$C_1 = -\frac{\dot{q}/m^2 K + \frac{h\theta_L}{Km} e^{mL} - \theta_0}{1 + e^{2mL}}$$

$$\text{or } C_1 = -\frac{[\dot{q}/m^2 K - \theta_0]\, e^{-mL} + h\theta_L/Km}{e^{mL} + e^{-mL}}$$

with a similar procedure & and using (a) $$C_2 = \frac{e^{mL}(\theta_0 - \dot{q}/m^2 K) + h\theta_L/Km}{e^{mL} + e^{-mL}}$$

Finally

$$\theta = \frac{e^{-mL}(\theta_0 - \dot{q}/m^2 K) - h\theta_L/Km}{e^{mL} + e^{-mL}} e^{mx} + \frac{e^{mL}(\theta_0 - \dot{q}/m^2 K) + \frac{h\theta_L}{Km}}{e^{mL} + e^{-mL}} e^{-mx} + \frac{\dot{q}}{m^2 K}$$

or using $$\cosh mL = \frac{e^{mL} + e^{-mL}}{2}$$

$$\sinh mL = \frac{e^{mL} - e^{-mL}}{2}$$

Problem 1 / Cont'd.

You can rewrite this equation

$$\theta = \frac{\left[\theta_o - \dot{q}/m^2k\right] e^{-mL} e^{mx} + \left[\theta_o - \dot{q}/m^2k\right] e^{mL} e^{-mx}}{2\cosh(mL)}$$

$$+ \frac{h\theta_L/km \left[-e^{mx} + e^{-mx}\right]}{2\cosh(mL)} + \frac{\dot{q}}{km^2}$$

or

$$\theta = \frac{\left[\theta_o - \dot{q}/m^2k\right] \overbrace{\left(e^{-m(L-x)} + e^{m(L-x)}\right)}^{\cancel{2}\cosh(m(L-x))}}{\cancel{2}\cosh(mL)}$$

$$+ \frac{h\theta_L/km \left[-\cancel{2}\sinh mx\right]}{\cancel{2}\cosh(mL)} + \frac{\dot{q}}{km^2}$$

or

$$\theta = \frac{\left[\theta_o - \dot{q}/m^2k\right] \cosh(m(L-x)) - \frac{h\theta_L}{km}\sinh(mx)}{\cosh(mL)} + \dot{q}/km^2$$

Ans.

(b) The total heat loss is due to convection and can be found from

$$dq_{conv.} = h\, dA_s\, \theta \rightarrow q_{conv.} = q_{loss} = \int h A_s \theta$$

we need to include heat transfer from the tip too which is $q_{tip} = h A_{tip} (\theta_L)$

So

$$q_{loss} = \int_0^L h\,dA_s\,\theta + h A_{tip}\,\theta_L$$

$$= \int_0^L h\,(\overbrace{P\,dx}^{dA_s})\,\theta + h A_{Tip}\,\theta_L \qquad A_{conduction} = A_{cross\ section}$$

$$= Ph \int_0^L \theta\,dx + h A_c\,\theta_L$$

$$= P.h \int_0^L \left(\frac{[\theta_0 - \dot{q}/m^2K]\cosh(m(L-x)) - \frac{h\theta_L}{km}\sinh(mx)}{\cosh(mL)} + \dot{q}/km^2 \right) dx$$

$$+ h A_c\,\theta_L$$

it becomes

$$q_{loss} = \frac{hP}{m}\left[-\frac{h\theta_L}{KL} + [\theta_0 - \dot{q}/m^2K]\tanh(mL) + \frac{h\theta_L}{Km\cosh mL} \right] + \dot{q}AL + hA\theta_L$$

Ans.

(C)

$$\left.\frac{dT}{dx}\right|_{x=0} = 0 \rightarrow \left.\frac{d\theta}{dx}\right|_{x=0} = 0$$

$$\theta = c_1 e^{mx} + c_2 e^{-mx} + \dot{q}/m^2K$$

$$\rightarrow c_1 m - mc_2 = 0$$

$$\rightarrow c_1 = c_2 = \frac{1}{2}\left(\theta_0 - \frac{\dot{q}A_1}{hp}\right)$$

and $\dot{q} = hP/A\left[\theta_0 + \frac{h\theta_L}{Km\sinh(mL)}\right]$ Ans.

Problem 2.

(a) $Bi = \frac{hL_c}{k}$

$L_c = \frac{\forall}{A_s}$

$\forall = [39 \times 24 \times 12] - 15 \times 6 \times 24 - \frac{\pi}{4}(16)^2 \times 12 = 6.659 \times 10^{-6}\ m^3$

$A = 3 \times 24 \times 12 + 2 \times 15 \times 6 + 2 \times 24 \times 6 + 2 \times 39 \times 24$

$- 2 \times \frac{\pi}{4}(16)^2 + \pi \times 16 \times 12 = 3.405 \times 10^{-3}\ m^2$

$\rightarrow L_c = \frac{6.659 \times 10^{-6}}{3.405 \times 10^{-3}} = 1.9556 \times 10^{-3} = 1.95\,mm$

$\rightarrow B_i = \frac{25 \times 0.00195}{18.9} = 2.58 \times 10^{-3} < 0.1$ lumped cap. is valid

(b) $F_o = \frac{\alpha t}{L_c^2} = \frac{\left(\frac{k}{\rho c_p}\right) t}{L_c^2} = \frac{\left(\frac{18.9}{7978 \times 559}\right) t}{(1.9556 \times 10^{-3})^2} = 1.1081 \times t$

and

$\frac{\theta}{\theta_i} = \exp(-B_i \times F_o)$

or $\frac{54 - 30}{600 - 30} = \exp(-2.58 \times 10^{-3} \times 1.1081 \times t)$

$\rightarrow t = 1107.97$ sec. $= 18.4$ min Ans.

Prob2./ cont'd.

(c) $q_{conv.} = h A_s (T - T_\infty)$

$= 25 \times 3.405 \times 10^{-3} \times (54 - 30) = 2.043 \, W$ Ans.

$q = -mc \frac{dT}{dt}$, $m = \rho V$

$\rightarrow \frac{dT}{dt} = \frac{-q}{mc} = \frac{-2.043}{7978 \times 6.65 \times 10^{-6} \times 559} = -6.88 \times 10^{-2} \, K/s$

Ans.