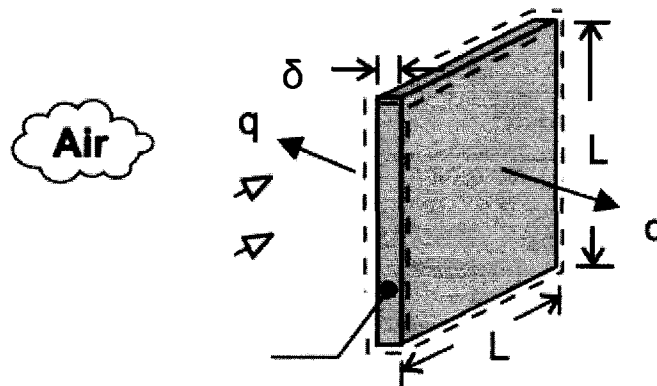


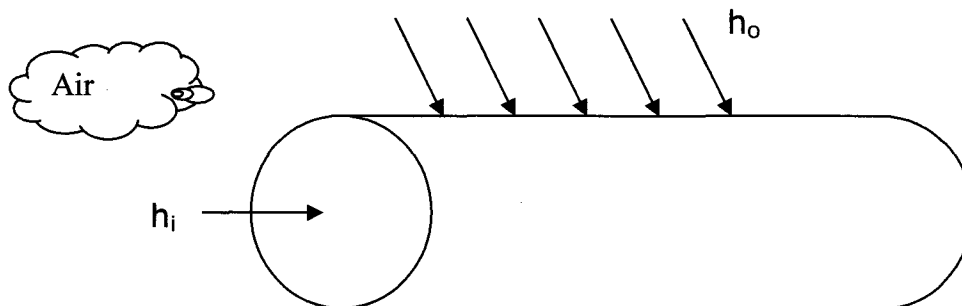
Problem 1 (10 points): Both sides of a steel plate of thickness 6 mm and length 1.5 m are cooled by atmospheric air of velocity $U_\infty=5$ m/s and $T_\infty=20^\circ\text{C}$.

- (a) If the initial temperature is 300°C , what is the rate of heat transfer from the plate? (3 points)
- (b) What is the corresponding rate of change of the plate temperature? (4 points)
- (c) If $U_\infty=15$ m/s and the initial temperature is 350°C what is the rate of heat transfer? (3 points)
- (d) (Bonus) If $U_\infty=3$ m/s and the constant heat flux from the plate surfaces is 1.2 w/m², what is the rate of heat transfer? (3 points)



Problem 2 (10 points): Air flows through a pipe of diameter 5cm with a convection heat transfer coefficient of $h_i=51$ w/m²K and air temperature of $T_i=25^\circ\text{C}$. If the thickness of the pipe is much smaller than its length and it is in cross flow with air at $T_o= 25$ and assuming $h_o=145$ w/m²k,

- (a) What is the wall temperature of the pipe? (2 points)
- (b) Compute the Nusselt number for the cross flow. (4 points)
- (c) Compute overall heat transfer coefficient (neglect conduction). (2 points)
- (d) Evaluate heat loss per unit length of the pipe. (2 points)



Problem 1:

Summer 2005, Heat Transfer

Quiz 3

For Stainless Steel

AISI 304 and

$T = 573K$

$T = 400 \rightarrow k = 16.6$

$T = 600 \rightarrow k = 19.8$

Table A1. $\frac{573-600}{400-600} = \frac{k-19.8}{16.6-19.8} \rightarrow k = 19.36 \text{ W/mK}$

$C = 560 \text{ J/kgK}$, $\rho = 7900 \text{ kg/m}^3$

$T_f = \frac{T_s + T_\infty}{2} = \frac{300 + 20}{2} = 160^\circ\text{C} = 433K$

For air, Table A4 $\rightarrow \nu = 30.4 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.0361 \text{ W/mK}$

$Pr = 0.688$

(a)

$Re_L = \frac{u_\infty L}{\nu} = \frac{5 \times 1.5}{30.4 \times 10^{-6}} = 2.47 \times 10^5 < 5 \times 10^5$

the flow is laminar

$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3} = 0.664 (2.47 \times 10^5)^{1/2} (0.688)^{1/3} = 291.2$

or $\bar{h} = \overline{Nu}_L \left(\frac{k}{L} \right) = 291.2 \left(\frac{0.0361}{1.5} \right) = 7.01 \text{ W/m}^2\text{K}$

the $q = \bar{h} A (T_s - T_\infty)$

$= \bar{h} (2 \times L \times L) (T_s - T_\infty)$

$= 2 \times 7.01 \times (1.5)^2 (300 - 20) = 8.829 \text{ kW}$

(b) $Bi = \frac{\bar{h} L_c}{k} = \frac{\bar{h} (V/A_s)}{k} = \left(\frac{\bar{h}}{k} \right) \left(\frac{L \times L \times \delta}{2(L \times L) + 4L\delta} \right) = \frac{\bar{h}}{k} \frac{\delta}{2}$

$L\delta \ll L^2 \rightarrow L_c = \frac{\delta}{2}$

$= \frac{7}{19.36} \times \frac{6 \times 10^{-3}}{2} = 1.08 \times 10^{-3} < 0.1$

It means that we can ignore temperature gradient inside the plate the the Energy Balance is:

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} - \dot{E}_{gen}$$

(Zero temp. gradient)

$$\text{So } -\bar{h} A_s (T_s - T_\infty) = \rho C V \frac{dT}{dt}$$

or $\underbrace{-\bar{h} (2L \times L)}_{q_{convection}} (T_s - T_\infty) = \rho C (L^2 \delta) \frac{dT}{dt}$

$$\text{or } \frac{dT}{dt} = \frac{-8.829 \times 10^3}{7900 \times 560 \times (1.5)^2 \times 6 \times 10^{-3}} = -0.1478 \text{ } ^\circ\text{C/s}$$

(c) if $U_\infty = 15 \text{ m/s}$, $T_i = 350^\circ\text{C}$

the $T_f = \frac{T_i + T_\infty}{2} = \frac{350 + 20}{2} = 185^\circ\text{C} = 458 \text{ K} \approx 450$

Table A4/Air $\nu = 32.39 \times 10^{-6} \text{ Pa.s}$, $k = 37.3 \times 10^{-3}$, $Pr = 0.686$

$$Re_L = \frac{U_\infty L}{\nu} = \frac{15 \times 1.5}{32.39 \times 10^{-6}} = 6.95 \times 10^5 > 5 \times 10^5$$

So flow at the end of the plate is turbulent.

we can check if we have mixed boundary layers,

$$Re_c = 5 \times 10^5 = \frac{U_\infty x_c}{\nu} = \frac{15 \times x_c}{32.39 \times 10^{-6}} \Rightarrow x_c = 1.079 \text{ m} < 1.5 \text{ m}$$

So mixed B.L.

So

$$\begin{aligned}\overline{Nu}_L &= (0.037 Re_L^{1/5} - 871) Pr^{1/3} \\ &= [0.037 \times (6.95 \times 10^5)^{1/5} - 871] (0.688)^{1/3} \\ &= 770.8\end{aligned}$$

$$\rightarrow \bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{770.8 \times 37.3 \times 10^{-3}}{1.5} = 19.17 \text{ W/m}^2\text{K}$$

the $q = \bar{h} (2L^2) (T_s - T_o)$

$$\begin{aligned}&= 19.17 (2 \times 1.5^2) (350 - 20) \\ &= 28.5 \text{ kW}\end{aligned}$$

(d) $U_\infty = 3 \text{ m/s}$, $q'' = 1.2 \text{ W/m}^2$ $\xrightarrow{\text{Simply}}$ $q = q'' A = 1.2 \times (1.5)^2 = 5.4 \text{ W}$

we don't know the surface temperature this time

so we need to do trial & error. let's say

$$T_s = 300 \xrightarrow{\text{Then}} T_f = 433 \text{ K} \rightarrow \text{we have the properties}$$

$$\text{now } Re_L = \frac{U_\infty L}{\nu} = \frac{3 \times 1.5}{30.4 \times 10^{-6}} = 148026 < 5 \times 10^5$$

the flow is entirely laminar then

(Eq. 7.53b) $\rightarrow \overline{Nu}_L = 0.680 Re_L^{1/2} Pr^{1/3} = 0.680 \times (148026)^{1/2} (0.688)^{1/3}$

$$\rightarrow \overline{Nu}_L = 230.96$$

Then

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = 230.96 \times \frac{0.0361}{1.5} = 5.558 \text{ W/m}^2\text{K}$$

now we have \bar{h} and we know q'' lets check

our guess for T_s is good or not,

$$q'' = \bar{h} (T_s - T_\infty) \rightarrow 1.2 = 5.558 \times (T_s - 20)$$

$$\rightarrow T_s = 20.22^\circ\text{C}$$

now lets use this T_s and repeat,

$$T_f = \frac{T_s + T_\infty}{2} = \frac{20.22 + 20}{2} = 20.11^\circ\text{C} = 293.11 \text{ K}$$

Table A-4 for air at 293K $\rightarrow \nu = 15.89 \times 10^{-6} \text{ Pa}\cdot\text{s}$
 $\approx 300\text{K} \rightarrow k = 26.3 \times 10^{-3} \text{ W/mK}$
 $Pr = 0.707$

$$Re_L = \frac{u_\infty L}{\nu} = \frac{3 \times 1.5}{15.89 \times 10^{-6}} = 283196 \approx 2.8 \times 10^5$$

Still laminar then

$$\overline{Nu}_L = 0.680 Re_L^{1/2} Pr^{1/3} = (0.680) (283196)^{1/2} (0.707)^{1/3} = 322$$

$$\text{then } \bar{h} = \overline{Nu}_L \frac{k}{L} = \frac{322.4 \times 26.3 \times 10^{-3}}{1.5} = 5.5878 \quad q = 5$$

$$\text{then } q'' = \bar{h} (T_s - T_\infty) \rightarrow T_s = 20.21^\circ\text{C} \quad \text{Ans.} \rightarrow q'' = 2L^2 \frac{q}{L^2} = 2L^2 \frac{5}{L^2}$$

Problem 2.

(5)

Energy Balance:

$$q''_{\text{conv},i} = q''_{\text{conv},o}$$

$$T_{\infty,i} = 25^{\circ}\text{C}, h_i = 51 \text{ W/m}^2\text{K}$$

$$T_{\infty,o} = 25^{\circ}\text{C}, h_o = 145 \text{ W/m}^2\text{K}$$

$$\rightarrow h_i (T_{\infty,i} - T_w) = h_o (T_w - T_{\infty,o})$$

$$\rightarrow 51 (25 - T_w) = 145 (T_w - 25)$$

$$(a) \rightarrow T_w = 25.0^{\circ}\text{C}$$

for outside convection

$$\text{Then } T_f = \frac{T_w + T_{\infty,o}}{2} = \frac{25 + 25}{2} = 25^{\circ}\text{C} = 298 \text{ K} \approx 300 \text{ K}$$

$$(b) \text{ Then } \nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 0.707$$

$$k = 23 \times 10^{-3} \text{ W/mK}$$

$$\text{Simply } \overline{Nu}_D = \frac{\overline{h}D}{k} = 276$$

or ↓

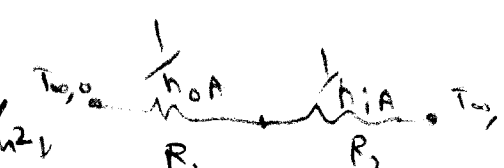
$$\text{Then } Re_D = \frac{U_{\infty} D}{\nu} = \frac{40 \times 0.05}{15.89 \times 10^{-6}} = 125865 \leftarrow$$

$$\text{Then Table 7.2} \rightarrow C = 0.027, m = 0.805$$

$$\text{Then } \overline{Nu}_D = C Re^{1/2} Pr^{1/3} = 0.027 \times Re^{0.805} Pr^{1/3}$$

$$(c) \text{ So } \overline{Nu}_D = 306.6$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{51} + \frac{1}{145} \rightarrow U = 38 \text{ W/m}^2\text{K}$$



$$(d) \quad \dot{Q} = U A_s (T_s - T_m) = 0 \quad \text{Ans.}$$