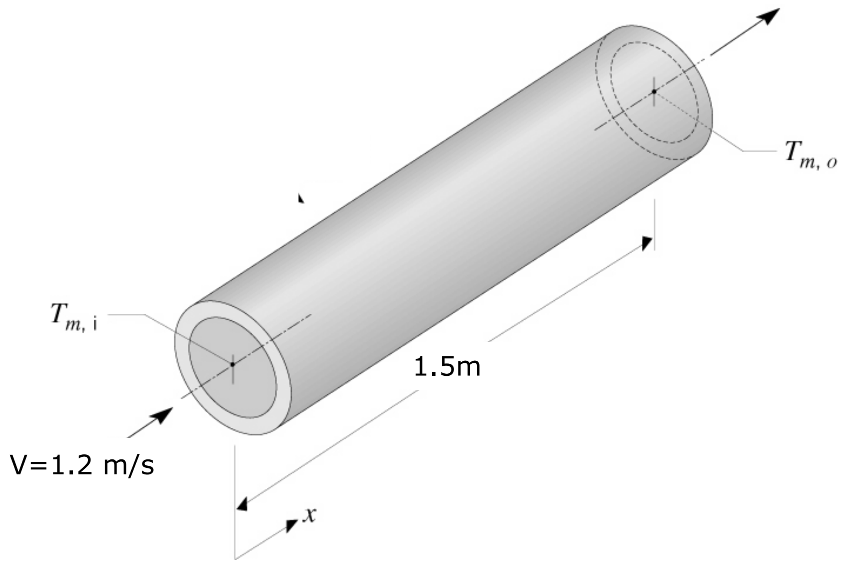
Int. To Heat Transfer (ME 411) Quiz 4 July 25, 2005
Name: (1 Hour)

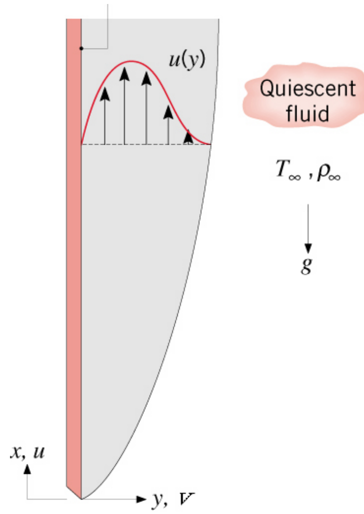
If solutions involve an iterative process multiple iterations are not necessary.

**Problem 1** (10 points): Nitrogen gas at atmospheric pressure and bulk inlet temperature of 27°C is heated in a tube with inner diameter of 2.5 cm at a constant surface temperature of 100°C. The bulk average velocity is 1.2 m/s and the tube is 1.5m long. Determine the net heat transfer rate to the nitrogen.




**Problem 2** (10 points): One side of a plate of dimensions 6x6x0.01m receives radiant heat flux from the sun of 1100 w/m$^2$. Assuming that 95 w/m$^2$ is conducted from the opposite side of the plate, estimate the *mid* surface temperature of the plate if the atmospheric air is at $T_\infty$=20°C, and the plate is

(a) vertical, (10 points)
(b) **(Bonus)** inclined θ=45°. (3 points)



Problem 1

$T_{m_i} = T_i = 15^\circ C$ $d_i = 2.5\ cm$

$V = 1.2\ m/s$ $T_s = 100^\circ C$

$L = 1.5\ m$

The exit temperature is unknown so we need to guess some value for it, and the correct our guess.

Trial 1: $T_{m,o} = 53^\circ C \longrightarrow \bar{T}_m = \frac{27+53}{2} = 40^\circ C = 313\ K$

Table A4 (Nitrogen): $\rho = 1.105\ kg/m^3$, $\nu = 16.94 \times 10^{-6}\ m^2/s$
$C_p = 1.042 \times 10^3\ J/kgK$, $P_r = 0.710$
$K = 0.0271\ W/mK$
$\mu = 18.37 \times 10^{-6}\ kg/ms$, $\mu_s = 20.86 \times 10^{-6}\ \frac{kg}{m\,s}$
at $T_s = 100^\circ C$

Check the flow regime

$$Re_D = \frac{DV}{\nu} = \frac{0.025 \times 1.2}{16.94 \times 10^{-6}} = 1771 < 2300 \quad \text{Laminar}$$

now lets check if it is fully developed or not

Velocity: $X = 0.05\, D\, Re_D = 0.05 \times 0.025 \times 1771 = 2.215\ m > 1.5\ m$

Temperature: $X_t = 0.05\, D\, Re_D\, P_r = 0.05 \times 0.025 \times 1771 \times 0.71 = 1.572\ m > 1.5\ m$

So both velocity & temperature profiles are still developing, using the Eq. 8.57 for entry region we have

$$\overline{Nu_D} = 1.86 \left(\frac{Re_D\, P_r}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

So
$$\overline{Nu_D} = 1.86\left(\frac{1771 \times 0.710}{60}\right)^{1/3}\left(\frac{18.37}{20.86}\right)^{0.14} = 5.037$$

Then
$$\bar{h} = \overline{Nu_D}\,\frac{K}{D} = 5.037 \times \frac{0.0271}{0.025} = 5.46\ W/m^2K$$

Now we can calculate the $T_{m,o}$ using Eq. 8.42

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(\frac{-PL}{\dot{m} c_p}\bar{h}\right) \quad \text{for } T_s = \text{const.}$$

where $\dot{m} = \rho V A = \rho V\left(\pi \frac{D^2}{4}\right) = 1.105 \times 1.2 \times \left(3.1415 \times \frac{0.025^2}{4}\right)$

$\rightarrow \dot{m} = 6.51 \times 10^{-4}\ Kg/s$

Now we have
$$\frac{100 - T_{m,o}}{100 - 27} = \exp\left(\frac{-\overbrace{\pi(0.025)}^{P} \times 1.5 \times 5.46}{6.51 \times 10^{-4} \times 1041.5}\right)$$

This gives $T_{m,o} = 71.7\,^\circ C$ This is the corrected outlet temperature

to obtain a better result we need to iterate once more,

Trial 2; $T_{m,o} = 71.7\,^\circ C \longrightarrow \bar{T}_m = \frac{27 + 71.7}{2} = 49.4\,^\circ C \simeq 50\,^\circ C = 323K$

Table A4 (Nitrogen): $\rho = 1.076\ Kg/m^3$ , $Pr = 0.708$
$C_p = 1.0419 \times 10^3\ J/Kg\,K$ ,
$\nu = 17.95 \times 10^{-6}\ m^2/s$ ,
$K = 0.0278\ pa.s$ , $\mu = 18.79 \times 10^{-6}\ Pa.s$

Again check the flow regime

$$Re_D = \frac{DV}{\nu} = \frac{0.025 \times 1.2}{17.95 \times 10^{-6}} = 1671 < 2300 \text{ laminar}$$

now $\overline{Nu_D} = 1.86 \left( \frac{1671 \times 0.708}{60} \right)^{1/3} \left( \frac{18.79 \times 10^{-6}}{20.86 \times 10^{-6}} \right)^{0.14}$

$$\rightarrow \overline{Nu_D} = 4.95 \rightarrow \bar{h} = \frac{K}{D} \overline{Nu_D} = \frac{0.0278}{0.025} (4.95)$$

$$\rightarrow \bar{h} = 5.504 \ W/m^2K$$

and

$$\dot{m} = \rho A V = 1.076 \times \pi \times \frac{0.025^2}{4} \times 1.2 = 6.33 \times 10^{-4} \ kg/s$$

Then

$$\frac{100 - T_{m,o}}{100 - 27} = \exp\left( \frac{-\pi \times 0.025 \times 1.5 \times 5.504}{6.33 \times 10^{-4} \times 1041.9} \right)$$

Then

$$\boxed{T_{m,o} = 73 \ ^\circ C} \quad \text{Ans.}$$

So this is sufficient for the $T_{m,o}$ evaluation

now to find the heat transfer we need the log-mean temperature.

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} = \frac{(100 - 73) - (100 - 27)}{\ln[(100 - 73)/(100 - 27)]} = 46.5^\circ C$$

$$\rightarrow q_{conv.} = \bar{h} A_s \Delta T_{lm} = 5.504 \times \underbrace{\pi \times 0.025 \times 1.5}_{A_s = \pi D L} \times 46.5^\circ C$$

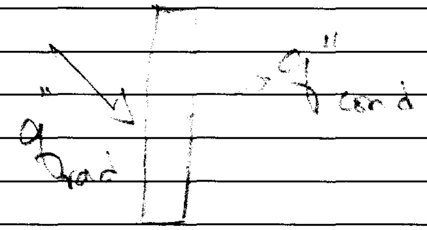
$$\rightarrow q_{conv.} = 30.2 \ W$$

Problem 2.

$L = W = 6\ m$

$\delta = 0.01\ m$

$q''_{rad} = 1100\ W/m^2$




$q''_{cond.} = 95\ W/m^2$

$T_{s,L/2} = T_s(x = \frac{L}{2}) = ?$

Start with Energy Balance.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} - \cancel{\dot{E}_{gen}}^{\,0}$$

$$\rightarrow q''_{rad} - q''_{cond.} = q''_{system} = q''$$

$$\rightarrow q'' = 1100 - 95 = 1005\ W/m^2$$

We can use equation 9.26 but need to use $T_{s,L/2}$ for the properties and we don't have this temperature, so we need to guess.

Trial 1: $T_{s,L/2} = 75^\circ C \longrightarrow T_f = \frac{75 + 20}{2} = 320.5\ K$

Table A4 (air): $\alpha = 25 \times 10^{-6}\ m^2/s$, $k = 28 \times 10^{-3}\ W/mK$

$\nu = 17.95 \times 10^{-6}\ Pa.s$

$Pr = 0.7$

$\beta = \frac{1}{T_f} = 3.12 \times 10^{-3}\ K^{-1}$

Now

$$Ra_L = \frac{g\beta(T_{s,L/2} - T_\infty)L^3}{\nu\alpha} = \frac{9.81 \times 3.12\times10^{-3}(75-20)\times 6^3}{17.95\times10^{-6}\times 25\times10^{-6}}$$

$$\rightarrow Ra_L = 8.1\times10^{11} > 10^9 \quad \text{Turbulent}$$

use Eq. 9.26

$$\overline{Nu_L} = \left[8.25 + \frac{0.387\,Ra_L^{1/6}}{\left(1+\left(\frac{0.492}{Pr}\right)^{9/16}\right)^{8/27}}\right]^2$$

$$\rightarrow \overline{Nu_L} = 1563.3 \rightarrow \bar{h} = \overline{Nu_L}\frac{K}{L} = 1563.3\times\frac{28\times10^{-3}}{6}$$

$$\rightarrow \bar{h} = 7.296 \ W/m^2K$$

Now we can find the correct $T_{s,L/2}$,

$$q'' = \bar{h}\,\Delta T_{L/2} = \bar{h}\left(T_{s,L/2} - T_\infty\right)$$

$$\rightarrow 1005 = 7.296\left(T_{s,L/2} - 20\right) \rightarrow T_{s,L/2} = 157.75\ ^\circ C$$

Now lets correct the results using this Temp.

$$T_m = \frac{T_{s,L/2} + T_\infty}{2} = \frac{157.75+20}{2} = 88.9^\circ C = 361.9\ K$$

Table A4 / air $\rightarrow$ $K = 0.0313\ W/mK$ , $\beta = 2.725\times10^{-3}\ K^{-1}$

$\nu = 22.8\times10^{-6}\ m^2/s$

$\alpha = 32.8\times10^{-6}\ m^2/s$

$Pr = 0.697$

check $Ra_L$ again

$$Ra_L = \frac{9.81 \times 2.725 \times 10^{-3} \times (157.75 - 20) \times 6^3}{22.8 \times 10^{-6} \times 32.8 \times 10^{-6}} = 1.064 \times 10^{12} > 10^9$$

Turbulent

now Eq. (9.26)

$$\overline{Nu_L} = \left[ 0.825 + \frac{0.387\,(1.064 \times 10^{12})^{1/6}}{\left[1 + \left(\frac{0.492}{0.697}\right)^{9/16}\right]^{8/27}} \right]^2$$

$$\rightarrow \overline{Nu_L} = 1679.50 \longrightarrow \bar{h} = \overline{Nu_L}\,\frac{k}{L} = 1679.50 \times \frac{0.0313}{6}$$

$$\rightarrow \bar{h} = 8.76 \ W/m^2 K$$

and

$$q'' = \bar{h}\,\Delta T_{L/2} = \bar{h}\,(T_{s,L/2} - T_\infty)$$

$$\rightarrow 1005 = 8.76\,(T_{s,L/2} - 20)$$

$$\rightarrow \boxed{T_{s,L/2} = 134.71\ ^\circ C}$$ for Vertical Wall

Bonus:

in $Ra_L = \frac{g\beta \Delta T L^3}{\nu \alpha}$ replace "$g$" with "$g\cos\theta$"

So $Ra_L = \frac{\beta g \cos\theta\, \Delta T L^3}{\nu \alpha}$ and repeat the problem.