Chapter 9 – Homework
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Chapter 9 • Compressible Flow

a molecular weight of 18, estimate (a) the exit gas temperature; (b) the mass flow; and (c) the thrust generated by the rocket.

NOTE: Sorry, we forgot to give the exit velocity, which is 1600 m/s.

Solution: (a) From Eq. (9.3), estimate $R_{\text{gas}}$ and hence the gas exit temperature:

$$R_{\text{gas}} = \frac{A}{M} = \frac{8314}{18} = 462 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

hence $T_{\text{exit}} = \frac{p}{R \rho} = \frac{54000}{462(0.15)} \approx 779 \text{ K} \quad \text{Ans. (a)}$

(b) The mass flow follows from the velocity which we forgot to give:

$$\dot{m} = \rho AV = \left(0.15 \frac{\text{kg}}{\text{m}^3}\right) \frac{\pi}{4} (0.45)^2 (1600) \approx 38 \frac{\text{kg}}{\text{s}} \quad \text{Ans. (b)}$$

(c) The thrust was derived in Problem 3.68. When $p_{\text{exit}} = p_{\text{ambient}}$, we obtain

$$\text{Thrust} = \rho_e A_e V_e^2 = \dot{m} V_e = 38(1600) = 61,100 \text{ N} \quad \text{Ans. (c)}$$

9.10 A certain aircraft flies at the same Mach number regardless of its altitude. Compared to its speed at 12000-m Standard Altitude, it flies 127 km/h faster at sea level. Determine its Mach number.

Solution: At sea level, $T_1 = 288.16 \text{ K}$. At 12000 m standard, $T_2 = 216.66 \text{ K}$. Then

$$a_1 = \sqrt{kRT_1} = \sqrt{1.4(287)(288.16)} = 340.3 \frac{\text{m}}{\text{s}}; \quad a_2 = \sqrt{kRT_2} = 295.0 \frac{\text{m}}{\text{s}}$$

Then $\Delta V_{\text{plane}} = Ma(a_2 - a_1) = Ma(340.3 - 295.0) = [127 \text{ km/h}] = 35.27 \text{ m/s}$

Solve for $Ma = \frac{35.27}{45.22} \approx 0.78 \quad \text{Ans.}$

9.11 At 300°C and 1 atm, estimate the speed of sound of (a) nitrogen; (b) hydrogen; (c) helium; (d) steam; and (e) uranium hexafluoride $^{238}\text{UF}_6$ ($k \neq 1.06$).

Solution: The gas constants are listed in Appendix Table A.4 for all but uranium gas (e):

(a) nitrogen: $k = 1.40$, $R = 297$, $T = 300 + 273 = 573 \text{ K}$:

$$a = \sqrt{kRT} = \sqrt{1.40(297)(573)} = 488 \text{ m/s} \quad \text{Ans. (a)}$$

(b) hydrogen: $k = 1.41$, $R = 4124$, $a = \sqrt{1.41(4124)(573)} = 1825 \text{ m/s} \quad \text{Ans. (b)}$

(c) helium: $k = 1.66$, $R = 2077$: $a = \sqrt{1.66(2077)(573)} = 1406 \text{ m/s} \quad \text{Ans. (c)}$
(d) steam: \( k = 1.33, \ R = 461; \ \ a = \sqrt{1.33(461)(573)} = \textbf{593 m/s} \) \ Ans. (d)

(e) For uranium hexafluoride, we need only to compute \( R \) from the molecular weight:

\[ \text{UF}_6: \ M = 238 + 6(19) = 352, \quad : \ R = \frac{8314}{352} = 23.62 \, \text{m}^2/\text{s}^2 \cdot \text{K} \]

then \( a = \sqrt{1.06(23.62)(573)} \approx \textbf{120 m/s} \) \ Ans. (e)

9.12 Assume that water follows Eq. (1.19) with \( n \approx 7 \) and \( B \approx 3000 \). Compute the bulk modulus (in kPa) and the speed of sound (in m/s) at (a) 1 atm; and (b) 1100 atm (the deepest part of the ocean). (c) Compute the speed of sound at 20°C and 9000 atm and compare with the measured value of 2650 m/s (A. H. Smith and A. W. Lawson, J. Chem. Phys., vol. 22, 1954, p. 351).

**Solution:** We may compute these values by differentiating Eq. (1.19) with \( k \approx 1.0: \)

\[ \frac{p}{p_a} = (B+1)(\rho/\rho_a)^n - B; \quad \text{Bulk modulus} \quad K = \rho \frac{dp}{d\rho} = n(B+1)p_a(\rho/\rho_a)^n, \quad a = \sqrt{K/\rho} \]

We may then substitute numbers for water, with \( p_a = 101350 \, \text{Pa} \) and \( \rho_a = 998 \, \text{kg/m}^3 \):

(a) at 1 atm: \( K_{\text{water}} = 7(3001)(101350)(1)^7 = \textbf{2.129E9 Pa} \) (21007 atm) \ Ans. (a)

speed of sound \( a_{\text{water}} = \sqrt{K/\rho} = \sqrt{2.129E9/998} \approx \textbf{1460 m/s} \) \ Ans. (a)

(b) at 1100 atm: \( \rho = 998 \left( \frac{1100+3000}{3001} \right)^{1/7} = 998(1.0456) \approx 1044 \, \text{kg/m}^3 \)

\[ K = K_{\text{atm}}(1.0456)^7 = (2.129E9)(1.3665) = \textbf{2.91E9 Pa} \] \ (28700 atm) \ Ans. (b)

\[ a = \sqrt{K/\rho} = \sqrt{2.91E9/1044} \approx \textbf{1670 m/s} \] \ Ans. (b)

(c) at 9000 atm: \( \rho = 998 \left( \frac{9000+3000}{3001} \right)^{1/7} = 1217 \, \text{kg/m}^3; \quad \text{K} = K_a \left( \frac{1217}{998} \right)^7, \]

or: \( K = 8.51E9 \, \text{Pa}, \quad a = \sqrt{K/\rho} = \sqrt{8.51E9/1217} \approx \textbf{2645 m/s} \) (within 0.2%) \ Ans. (c)

9.13 Assume that the airfoil of Prob. 8.84 is flying at the same angle of attack at 6000 m standard altitude. Estimate the forward velocity, in mi/h, at which supersonic flow (and possible shock waves) will appear on the airfoil surface.

**Solution:** At 6000 m, from Table A.6, \( a = 316.5 \, \text{m/s} \). From the data of Prob. 8.84, the highest surface velocity is about \( 1.29U_{\infty} \) and occurs at about the quarter-chord point.
\[
p/p_a = 80.4 = 3001(\rho/1025)^7 - 3000, \quad \text{solve } \rho \approx 1029 \text{ kg/m}^3
\]
\[a = \sqrt{n(B+1)p_a(\rho/\rho_a)^7/\rho} = \sqrt[7]{3001}(101350)(1029/1025)^7/1029 = 1457 \text{ m/s}
\]

Hardly worth the trouble: One-way distance \(a \Delta t/2 = 1457(15/2) \approx 10900 \text{ m}\). \(\text{Ans.}\)

9.18 Race cars at the Indianapolis Speedway average speeds of 185 mi/h. After determining the altitude of Indianapolis, find the Mach number of these cars and estimate whether compressibility might affect their aerodynamics.

**Solution:** Rush to the Almanac and find that Indianapolis is at 220 m altitude, for which Table A.6 predicts that the standard speed of sound is 339.4 m/s = 759 mi/h. Thus the Mach number is
\[
Ma_{racer} = V/a = 185 \text{ mph}/759 \text{ mph} = 0.24 \quad \text{Ans.}
\]
This is less than 0.3, so the Indianapolis Speedway need not worry about compressibility.

9.19 The Concorde aircraft flies at \(Ma \approx 2.3\) at 11-km standard altitude. Estimate the temperature in \(^\circ\text{C}\) at the front stagnation point. At what Mach number would it have a front stagnation point temperature of 450\(^\circ\text{C}\)?

**Solution:** At 11-km standard altitude, \(T = 216.66 \text{ K}\), \(a = \sqrt{(kRT)} = 295 \text{ m/s}\). Then
\[
T_{\text{nose}} = T_o = T\left(1 + \frac{k-1}{2} Ma^2\right) = 216.66\left[1 + 0.2(2.3)^2\right] = 446 \text{ K} \approx 173\text{\(^\circ\text{C}\)} \quad \text{Ans.}
\]
If, instead, \(T_o = 450\text{\(^\circ\text{C}\)} = 723 \text{ K}\) = 216.66\left[1 + 0.2 Ma^2\right], solve \(Ma \approx 3.42 \quad \text{Ans.}\)

9.20 A gas flows at \(V = 200 \text{ m/s}, p = 125 \text{ kPa},\) and \(T = 200\text{\(^\circ\text{C}\)}\). For (a) air and (b) helium, compute the maximum pressure and the maximum velocity attainable by expansion or compression.

**Solution:** Given \((V, p, T)\), we can compute \(Ma, T_o\) and \(p_o\) and then \(V_{\text{max}} = \sqrt{(2c_p T_o)}:\)

(a) air:
\[
Ma = \frac{V}{\sqrt{kRT}} = \frac{200}{\sqrt{1.4(287)(200+273)}} = \frac{200}{436} = 0.459
\]
Then
\[
p_{\text{max}} = p_o = p\left(1 + \frac{k-1}{2} Ma^2\right)^{k/(k-1)} = 125\left[1 + 0.2(0.459)^2\right]^{3.5} = 144 \text{ kPa} \quad \text{Ans. (a)}
\]
\[
T_o = (200+273)[1 + 0.2(0.459)^2] = 493 \text{ K}, \quad V_{\text{max}} = \sqrt{2(1005)(493)} = 995 \text{ m/s} \quad \text{Ans. (a)}
\]
(b) For helium, \( k = 1.66, \ R = 2077 \ m^2/s^2 \cdot K, \ c_p = kR/(k - 1) = 5224 \ m^2/s^2 \cdot K \). Then

\[
\begin{align*}
Ma &= 200/\sqrt{1.66(2077)(473)} = 0.157, \quad p_o = 125[1 + 0.33(0.157)^2]^{0.66} \approx 128 \ kPa \\
T_o &= 473[1 + 0.33(0.157)^2] = 477 \ K, \quad V_{max} = \sqrt{2(5224)(477)} = 2230 \ m/s \quad \text{Ans. (b)}
\end{align*}
\]

9.21 CO\(_2\) expands isentropically through a duct from \( p_1 = 125 \ kPa \) and \( T_1 = 100^\circ C \) to \( p_2 = 80 \ kPa \) and \( V_2 = 325 \ m/s \). Compute (a) \( T_2 \); (b) \( Ma_2 \); (c) \( T_o \); (d) \( p_o \); (e) \( V_1 \); and (f) \( Ma_1 \).

**Solution:** For CO\(_2\), from Table A.4, take \( k = 1.30 \) and \( R = 189 \ J/kg \cdot K \). Compute the specific heat:

\[
c_p = kR/(k - 1) = 1.3(189)/(1.3 - 1) = 819 \ J/kg \cdot K
\]

The results follow in sequence:

(a) \( T_2 = T_1(p_2/p_1)^{(k-1)/k} = (373 \ K)(80/125)^{(1.3-1)/1.3} = 336 \ K \quad \text{Ans. (a)} \)

(b) \( a_2 = \sqrt{kRT_2} = \sqrt{(1.3)(189)(336)} = 288 \ m/s, \ Ma_2 = V_2/a_2 = 325/288 = 1.13 \quad \text{Ans. (b)} \)

(c) \( T_o = T_o = T_2 \left[ 1 + \frac{k-1}{2} \left( \frac{V_2}{c_p} \right)^2 \right] = (336) \left[ 1 + \frac{0.3}{2}(1.13)^2 \right] = 401 \ K \quad \text{Ans. (c)} \)

(d) \( p_o = p_o/p_2 = p_2 \left[ 1 + \frac{k-1}{2} \left( \frac{V_2}{c_p} \right)^2 \right]^{1.3/(1.3-1)} = (80) \left[ 1 + \frac{0.3}{2}(1.13)^2 \right]^{1.3/0.3} = 171 \ kPa \quad \text{Ans. (d)} \)

(e) \( T_o = 401 \ K = T_1 + \frac{V_1^2}{2c_p^2} = 373 + \frac{V_1^2}{2(819)}, \quad \text{Solve for} \ V_1 = 214 \ m/s \quad \text{Ans. (e)} \)

(f) \( a_1 = \sqrt{kRT_1} = \sqrt{(1.3)(189)(373)} = 303 \ m/s, \ Ma_1 = V_1/a_1 = 214/303 = 0.71 \quad \text{Ans. (f)} \)

9.22 Given the pitot stagnation temperature and pressure and the static-pressure measurements in Fig. P9.22, estimate the air velocity \( V \), assuming (a) incompressible flow and (b) compressible flow.

**Solution:** Given \( p = 80 \ kPa, \ p_o = 120 \ kPa, \) and \( T = 100^\circ C = 373 \ K \). Then

\[
\rho_o = \frac{p_o}{RT_o} = \frac{120000}{287(373)} = 1.12 \ kg/m^3
\]
(a) ‘Incompressible’:

\[
\rho = \rho_o, \quad V = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2(120000 - 80000)}{1.12}} \approx 267 \text{ m/s} \quad (7\% \text{ low}) \quad \text{Ans. (a)}
\]

(b) Compressible: \(T = T_o(p/p_o)^{(k-1)/k} = 373(80/120)^{0.4/1.4} = 332 \text{ K}\). Then \(T_o = 373 \text{ K} = T + V^2/2c_p = 332 + V^2/[2(1005)], \) solve for \(V = 286 \text{ m/s}\). \(\text{Ans. (b)}\)

**9.23** A large rocket engine delivers hydrogen at 1500°C and 3 MPa, \(k = 1.41, R = 4124 \text{ J/kg·K}\), to a nozzle which exits with gas pressure equal to the ambient pressure of 54 kPa. Assuming isentropic flow, if the rocket thrust is 2 MN, estimate (a) the exit velocity; and (b) the mass flow of hydrogen.

**Solution:** Compute \(c_p = kR/(k-1) = 14180 \text{ J/kg·K}\). For isentropic flow, compute

\[
\rho_o = \frac{p_o}{RT_o} = \frac{3E6}{4124(1773)} = 0.410 \text{ kg/m}^3, \quad \therefore \rho_e = \rho_o \left( \frac{p_e}{p_o} \right)^{1/k} = 0.410 \left( \frac{54E3}{3E6} \right)^{1/1.41} = 0.0238 \text{ kg/m}^3
\]

\[
T_e = \frac{54000}{4124(0.0238)} = 551 \text{ K}, \quad T_o = 1773 = 551 + \frac{V_e^2}{2(14180)},
\]

Solve \(V_{exit} = 5890 \text{ m/s} \quad \text{Ans. (a)}\)

From Prob. 3.68, \(\text{Thrust} = 2E6 \text{ N} = \dot{m}V_e = \dot{m}(5890), \) solve \(\dot{m} = 340 \text{ kg/s} \quad \text{Ans. (b)}\)

**9.24** For low-speed (nearly incompressible) gas flow, the stagnation pressure can be computed from Bernoulli’s equation

\[
p_0 = p + \frac{1}{2} \rho V^2
\]

(a) For higher subsonic speeds, show that the isentropic relation (9.28a) can be expanded in a power series as follows:

\[
p_0 \approx p + \frac{1}{2} \rho V^2 \left( 1 + \frac{1}{4} \text{Ma}^2 + \frac{2-k}{24} \text{Ma}^4 + \cdots \right)
\]

(b) Suppose that a pitot-static tube in air measures the pressure difference \(p_0 - p\) and uses the Bernoulli relation, with stagnation density, to estimate the gas velocity. At what Mach number will the error be 4 percent?
9.26 Show that for isentropic flow of a perfect gas if a pitot-static probe measures \( p_0 \), \( p \), and \( T_0 \), the gas velocity can be calculated from

\[
V^2 = 2c_p T_0 \left[ 1 - \left( \frac{p}{p_0} \right)^{(k-1)/k} \right]
\]

What would be a source of error if a shock wave were formed in front of the probe?

**Solution:** Assuming isentropic flow past the probe,

\[ T = T_0 \left( \frac{p}{p_0} \right)^{(k-1)/k} = T_0 - \frac{V^2}{2c_p}, \quad \text{solve} \quad V^2 = 2c_p T_0 \left[ 1 - \left( \frac{p}{p_0} \right)^{(k-1)/k} \right] \]

Ans.

If there is a *shock wave* formed in front of the probe, this formula will yield the air velocity inside the shock, because the probe measures \( p_o \) inside the shock. The stagnation pressure in the outer stream is greater, as is the velocity outside the shock.

9.27 In many problems the sonic (*) properties are more useful reference values than the stagnation properties. For isentropic flow of a perfect gas, derive relations for \( \frac{p}{p^*} \), \( \frac{T}{T^*} \), and \( \frac{\rho}{\rho^*} \) as functions of the Mach number. Let us help by giving the density-ratio formula:

\[
\frac{\rho}{\rho^*} = \left[ 1 + \frac{k-1}{2} \left( \frac{\rho}{\rho_0} \right)^{(k-1)/2} \right]^{1/(k-1)}
\]

Solution: Simply introduce (and then cancel out) the stagnation properties:

\[
\frac{\rho}{\rho^*} = \frac{\rho}{\rho_0} \left( \frac{1 + \frac{k-1}{2} \left( \frac{\rho}{\rho_0} \right)^{(k-1)/2}}{1 + \frac{k-1}{2}} \right)^{-1/(k-1)} = \left[ 1 + \frac{k-1}{2} \left( \frac{\rho}{\rho_0} \right)^{(k-1)/2} \right]^{1/(k-1)} \quad \text{Ans.}
\]

\[
\frac{p}{p^*} = \frac{p}{p_0} \left[ \frac{k+1}{2 + (k-1) \left( \frac{\rho}{\rho_0} \right)^{(k-1)/2}} \right]^{(k-1)/2} \quad \text{and} \quad \frac{T}{T^*} = \frac{T}{T_0} \left[ \frac{k+1}{2 + (k-1) \left( \frac{\rho}{\rho_0} \right)^{(k-1)/2}} \right]^{(k-1)/2} \quad \text{Ans.}
\]

9.28 A large vacuum tank, held at 60 kPa absolute, sucks sea-level standard air through a converging nozzle of throat diameter 3 cm. Estimate (a) the mass flow rate; and (b) the Mach number at the throat.
Solution: For sea-level air take \( T_0 = 288 \text{ K}, \rho_0 = 1.225 \text{ kg/m}^3, \) and \( p_0 = 101350 \text{ Pa}. \) The pressure ratio is given, and we can assume isentropic flow with \( k = 1.4: \)

\[
\frac{p_e}{p_o} = \frac{60000}{101350} = (1 + 0.2Ma_e^2)^{-3.5}, \quad \text{solve } Ma_e = 0.899 \quad \text{Ans. (b)}
\]

We can then solve for exit temperature, density, and velocity, finally mass flow:

\[
\rho_e = \rho_o [1 + 0.2(0.899)^2]^{-2.5} \approx 0.842 \frac{\text{kg}}{\text{m}^3}, \quad T_e = \frac{p_e}{R\rho_e} = \frac{60000}{287(0.842)} \approx 248 \text{ K}
\]

\[
V_e = Ma_e a_e = 0.899[1.4(287)(248)]^{1/2} \approx 284 \frac{\text{m}}{\text{s}}
\]

Finally, \( \dot{m} = \rho_e A_e V_e = (0.842) \frac{\pi}{4} (0.03)^2 (284) \approx 0.169 \frac{\text{kg}}{\text{s}} \quad \text{Ans. (a)}
\]

9.29 Steam from a large tank, where \( T = 400^\circ \text{C} \) and \( p = 1 \text{ MPa}, \) expands isentropically through a small nozzle until, at a section of 2-cm diameter, the pressure is 500 kPa. Using the Steam Tables, estimate (a) the temperature; (b) the velocity; and (c) the mass flow at this section. Is the flow subsonic?

Solution: “Large tank” is code for stagnation values, thus \( T_0 = 400^\circ \text{C} \) and \( p_o = 1 \text{ MPa} \). This problem involves dogwork in the tables and well illustrates why we use the ideal-gas law so readily. Using \( k \approx 1.33 \) for steam, we find the flow is slightly supersonic:

Ideal-gas simplification:

\[
\frac{p_o}{p} = \frac{1000}{500} = 2.0 \approx \left[ 1 + \frac{1.33 - 1}{2} \right] Ma^{0.33},
\]

Solve \( Ma \approx 1.08 \)

That was quick. Instead, plow about in the S.I. Steam Tables, assuming constant entropy:

At \( T_0 = 400^\circ \text{C} \) and \( p_o = 1 \text{ MPa}, \) read \( s_o \approx 7481 \frac{\text{J}}{\text{kg} \cdot \text{K}} \) and \( h_o \approx 3.264 \text{E}6 \frac{\text{J}}{\text{kg}} \)

Then, at \( p = 0.5 \text{ MPa}, \) assuming \( s = s_o, \) read \( T \approx 304^\circ \text{C} \approx 577 \text{ K} \quad \text{Ans. (a)} \)

Also read \( h \approx 3.074 \text{E}6 \frac{\text{J}}{\text{kg}} \) and \( \rho \approx 1.896 \text{ kg/m}^3. \)
9.31 Air flows adiabatically through a duct. At one section, \( V_1 = 400 \text{ ft/s} \), \( T_1 = 200^\circ \text{F} \), and \( p_1 = 35 \text{ psia} \), while farther downstream \( V_2 = 1100 \text{ ft/s} \) and \( p_2 = 18 \text{ psia} \). Compute (a) \( M_2 \); (b) \( U_{\text{max}} \); and (c) \( p_{o2}/p_{o1} \).

**Solution:**
(a) Begin by computing the stagnation temperature, which is constant (adiabatic):

\[
T_o = T_1 + \frac{V^2}{2c_p} = (200 + 460) + \frac{(400)^2}{2(6010)} = 673^\circ \text{R} = T_2 + \frac{V^2}{2c_p}
\]

Then \( T_2 = 673 - \frac{(1100)^2}{2(6010)} = 573^\circ \text{R} \),

\[
M_2 = \frac{V_2}{a_2} = \frac{1100}{\sqrt{1.4(1717)(573)}} = \frac{1100}{1173} = 0.938 \quad \text{Ans. (a)}
\]

(b) \( U_{\text{max}} = \sqrt{2c_p T_o} = \sqrt{2(6010)(673)} \approx 2840 \text{ ft/s} \quad \text{Ans. (b)} \)

(c) We need \( M_1 = V_1/a_1 = 400/\sqrt{1.4(1717)(200 + 460)} = 400/1260 = 0.318 \)

Then \( p_{o1} = p_1 \left(1 + 0.2M_1^2\right)^{3.5} = 1.072p_1 = 37.53 \text{ psia} \)

and \( p_{o2} = p_2 \left(1 + 0.2M_2^2\right)^{3.5} = 1.763p_2 = 31.74 \text{ psia} \)

\[
\frac{p_{o2}}{p_{o1}} = \frac{31.74}{37.53} \approx 0.846 \quad \text{Ans. (c)}
\]

9.32 The large compressed-air tank in Fig. P9.32 exhausts from a nozzle at an exit velocity of 235 m/s. The mercury manometer reads \( h = 30 \text{ cm} \). Assuming isentropic flow, compute the pressure (a) in the tank and (b) in the atmosphere. (c) What is the exit Mach number?

**Solution:** The tank temperature = \( T_o = 30^\circ \text{C} = 303 \text{ K} \). Then the exit jet temperature is

\[
T_e = T_o - \frac{V_e^2}{2c_p} = 303 - \frac{(235)^2}{2(1005)} = 276 \text{ K}, \quad \therefore \ M_e = \frac{235}{\sqrt{1.4(287)(276)}} \approx 0.706 \quad \text{Ans. (c)}
\]

Then \( \frac{p_{\text{tank}}}{p_e} = \left(1 + 0.2M_e^2\right)^{3.5} = 1.395 \) and \( p_{\text{tank}} - p_e = (\rho_{\text{mercury}} - \rho_{\text{tank}})gh \)

Guess \( \rho_{\text{tank}} \approx 1.6 \text{ kg/m}^3 \), \( \therefore \ p_o - p_e \approx (13550 - 1.6)(9.81)(0.30) \approx 39900 \text{ Pa} \)

Solve the above two simultaneously for \( p_e \approx 101 \text{ kPa} \) and \( p_{\text{tank}} \approx 140.8 \text{ kPa} \quad \text{Ans. (a, b)} \)
(c) Assume $\rho = \rho_{o} = \rho_{\text{tire}}$, for how would we know $\rho_{\text{exit}}$ if we didn’t use compressible-flow theory? Then the incompressible Bernoulli relation predicts

$$\rho_{o} = \frac{p_{o}}{RT_{o}} = \frac{169120}{287(303)} = 1.945 \frac{\text{kg}}{\text{m}^{3}}$$

$$V_{e,\text{in}} = \sqrt{\frac{2\Delta p}{\rho_{o}}} = \sqrt{\frac{2(169120-100000)}{1.945}} = 267 \frac{\text{m}}{\text{s}} \text{ Ans. (c)}$$

This is 8% lower than the “exact” estimate in part (a).

9.43 Air flows isentropically through a duct with $T_{o} = 300^\circ \text{C}$. At two sections with identical areas of 25 cm$^{2}$, the pressures are $p_{1} = 120$ kPa and $p_{2} = 60$ kPa. Determine (a) the mass flow; (b) the throat area, and (c) $M_{a_{2}}$.

**Solution:** If the areas are the same and the pressures different, then section (1) must be subsonic and section (2) supersonic. In other words, we need to find where

$$p_{1}/p_{o} = \frac{120}{60} = 2.0 \text{ for the same } A_{i}/A^{*} = A_{2}/A^{*} \text{—search Table B.1 (isentropic)}$$

After laborious but straightforward iteration, $M_{a_{1}} = 0.729, \text{ } M_{a_{2}} = 1.32 \text{ Ans. (c)}$

$$A/A^{*} = 1.075 \text{ for both sections, } A^{*} = 25/1.075 = 23.3 \text{ cm}^{2} \text{ Ans. (b)}$$

With critical area and stagnation conditions known, we may compute the mass flow:

$$p_{o} = 120[1 + 0.2(0.729)^{2}]^{3.5} = 171 \text{ kPa and } T_{o} = 300 + 273 = 573 \text{ K}$$

$$\dot{m} = 0.6847p_{o}A^{*}/[RT_{o}]^{1/2} = 0.6847(171000)(0.00233)/[287(573)]^{1/2}$$

$$\dot{m} = 0.671 \frac{\text{kg}}{\text{s}} \text{ Ans. (a)}$$

9.44 In Prob. 3.34 we knew nothing about compressible flow at the time so merely assumed exit conditions $p_{2}$ and $T_{2}$ and computed $V_{2}$ as an application of the continuity equation. Suppose