9.57 Air flows from a tank through a nozzle into the standard atmosphere, as in Fig. P9.57. A normal shock stands in the exit of the nozzle, as shown. Estimate (a) the tank pressure; and (b) the mass flow.

Solution: The throat must be sonic, and the area ratio at the shock gives the Mach number:

\[
\frac{A_1}{A^*} = 1.4 = \frac{1 + 0.2M_a^2}{1.728M_a}.
\]

Solve \(M_a \approx 1.76\) upstream of the shock

Then \(p_2/p_1 \bigg|_{\text{shock}} = \frac{2.8(1.76)^2 - 0.4}{2.4} \approx 3.46,\) \(p_2 = 1\) atm, \(p_1 = \frac{101350}{3.46} \approx 29289\) Pa.

Thus \(p_{\text{tank}} = p_{o1} = 29289[1 + 0.2(1.76)^2]^{3.5} \approx 159100\) Pa \(\text{Ans. (a)}\)

Given that \(T_o = 100^\circ C = 373\) K and a critical throat area of 10 cm\(^2\), we obtain

\[
m = m_{\text{max}} = \frac{0.6847p_oA^*/\sqrt{RT_o}}{0.6847(159100)(0.001)/\sqrt{287(373)}} = 0.333 \frac{\text{kg}}{\text{s}}\ \text{Ans. (b)}
\]

9.58 Argon (Table A.4) approaches a normal shock with \(V_1 = 700\) m/s, \(p_1 = 125\) kPa, and \(T_1 = 350\) K. Estimate (a) \(V_2\), and (b) \(p_2\). (c) What pressure \(p_2\) would result if the same velocity change \(V_1\) to \(V_2\) were accomplished isentropically?

Solution: For argon, take \(k = 1.67\) and \(R = 208\) J/kg·K. Determine the Mach number upstream of the shock:

\[
a_1 = \sqrt{kRT_1} = \sqrt{1.67(208)(350)} \approx 349 \frac{\text{m}}{\text{s}}; \quad M_a = \frac{V_1}{a_1} = \frac{700}{349} \approx 2.01
\]

Then \(p_2/p_1 \bigg|_{\text{shock}} = \frac{2(1.67)(2.01)^2 - 0.67}{1.67 + 1} \approx 4.79,\) \(\text{or}\ p_2 = 4.79(125) = 599\) kPa \(\text{Ans. (b)}\)

and \(V_2/V_1 = \frac{0.67(2.01)^2 + 2}{2.67(2.01)^2} = 0.437,\) \(\text{or}\ V_2 = 0.437(700) = 306\) m/s \(\text{Ans. (a)}\)
9.63 Sea-level standard air is sucked into a vacuum tank through a nozzle, as in Fig. P9.63. A normal shock stands where the nozzle area is $2 \text{ cm}^2$, as shown. Estimate (a) the pressure in the tank; and (b) the mass flow.

Solution: The flow at the exit section (“3”) is subsonic (after a shock) therefore must equal the tank pressure. Work our way to 1 and 2 at the shock and thence to 3 in the exit:

$$p_{01} = 101350 \text{ Pa}, \quad A_1/A^* = 2.0, \quad \text{thus } Ma_1 \approx 2.1972, \quad p_1 = \frac{101350}{[1 + 0.2(2.2)^2]^{3.5}} \approx 9520 \text{ Pa}$$

$$p_2 = \frac{2.8(2.2)^2 - 0.4}{2.4} = 5.47, \quad \therefore \quad p_2 = 5.47(9520) = 52030 \text{ Pa}$$

Also compute $A_2^*/A_1^* \approx 1.59, \quad \text{or } A_2^* = 1.59 \text{ cm}^2$

Also compute $p_{02} = 101350/1.59 = 63800 \text{ Pa}$. Finally compute $A_2/A_2^* = 3/1.59 = 1.89$, read $Ma_3 = 0.327$, whence $p_3 = 63800/[1 + 0.2(0.327)^2]^{2/3.5} \approx 59200 \text{ Pa}$. Ans. (a).

With $T_o = 288 \text{ K}$, the (critical) mass flow $= 0.6847p_oA^*/\sqrt{(RT_o)} = 0.0241 \text{ kg/s}$. Ans. (b)

9.64 Air in a large tank at $100^\circ\text{C}$ and $150 \text{ kPa}$ exhausts to the atmosphere through a converging nozzle with a $5-\text{cm}^2$ throat area. Compute the exit mass flow if the atmospheric pressure is (a) $100 \text{ kPa}$; (b) $60 \text{ kPa}$; and (c) $30 \text{ kPa}$.

Solution: Choking occurs when $p_{\text{atmos}} < 0.5283p_{\text{tank}} = 79 \text{ kPa}$. Therefore the first case is not choked, the second two cases are. For the first case, with $T_o = 100^\circ\text{C} = 373 \text{ K}$,

(a) $\frac{p_o}{p_e} = \frac{150}{100} = 1.5 = (1 + 0.2Ma_e^2)^{3.5}$, solve $Ma_e = 0.784$, $T_e = \frac{373}{1 + 0.2(0.784)^2} = 332 \text{ K}$

and $a_e = \sqrt{1.4(287)(332)} = 365 \frac{\text{m}}{\text{s}}$, $V_e = 0.784(365) = 286 \text{ m/s}$,

and $\rho_e = p_e/RT_e = 1.05 \text{ kg/m}^3$, finally: $m = 1.05(0.0005)(286) = 0.150 \text{ kg/s}$ Ans. (a)

Both cases (b) and (c) are choked, with $p_{\text{atm}} \leq 79 \text{ kPa}$, and the mass flow is maximum and driven by tank conditions $T_o$ and $p_o$:

(b, c) $m = m_{\text{max}} = \frac{0.6847p_oA^*/\sqrt{(RT_o)}}{\sqrt{287(373)}} = \frac{0.6847(150000)(0.0005)}{\sqrt{287(373)}} \approx 0.157 \frac{\text{kg}}{\text{s}}$ Ans. (b, c)
of this shock is 473 K. Calculate (a) the temperature in the large tank; (b) the receiver pressure; and (c) the mass flow.

**Solution:** First find the Mach number just upstream of the shock and the temperature ratio:

\[
\frac{A_{\text{exit}}}{A^*} = \frac{2.2 \text{ cm}^2}{1.0 \text{ cm}^2} = 2.2 = \left(1 + 0.2Ma_1^2\right)^{3}, \quad \text{solve for } Ma_1 = 2.303
\]

Across the shock:

\[
\frac{T_2}{T_1} = \frac{473 \text{ K}}{243 \text{ K}} = \frac{[2 + 0.4(2.303)^2][2.8(2.303)^2 - 0.4]}{(2.4)^2(2.303)^2} = 1.95, \quad T_1 = 243 \text{ K}
\]

\[
T_{\text{tank}} = T_o = T_1\left(1 + 0.2Ma_1^2\right) = (243 \text{ K})[1 + 0.2(2.303)^2] = 500 \text{ K} \quad \text{Ans. (a)}
\]

Finally, compute the pressures just upstream and downstream of the shock:

\[
p_1 = p_o\left(1 + 0.2Ma_1^2\right)^{3.5} = (300 \text{ kPa})/[1 + 0.2(2.303)^2]^{3.5} = 23.9 \text{ kPa}
\]

\[
p_2 = p_{\text{receiver}} = \frac{p_1}{k+1}\left(2kMa_1^2 - k + 1\right) = \frac{23.9 \text{ kPa}}{(1.4 + 1)}[2(1.4)(2.303)^2 - 0.4] = 144 \text{ kPa} \quad \text{Ans. (b)}
\]

---

**9.86** Air enters a 3-cm diameter pipe 15 m long at \(V_1 = 73 \text{ m/s}, p_1 = 550 \text{ kPa}, \) and \(T_1 = 60^\circ \text{C}.\) The friction factor is 0.018. Compute \(V_2, p_2, T_2,\) and \(p_{02}\) at the end of the pipe. How much additional pipe length would cause the exit flow to be sonic?

**Solution:** First compute the inlet Mach number and then get \((fL/D)_1:\)

\[
a_1 = \sqrt{1.4(287)(60 + 273)} = 366 \frac{\text{m}}{\text{s}}, \quad Ma_1 = \frac{73}{366} = 0.20, \quad \text{read } \frac{(fL)}{D} = 14.53,
\]

for which \(p/p^* = 5.4554, \) \(T/T^* = 1.1905, \) \(V/V^* = 0.2182,\) and \(p_o/p_o^* = 2.9635\)

Then \( (fL/D)_2 = 14.53 - (0.018)(15)/(0.03) = 5.53, \) \( \text{read } Ma_2 \approx 0.295\)

At this new \(Ma_2,\) read \(p/p^* \approx 3.682, \) \(T/T^* \approx 1.179, \) \(V/V^* \approx 0.320, \) \(p_o/p_o^* \approx 2.067.\) Then

\[
V_2 = V_1 \frac{V_2/V^*}{V_1/V^*} = 73 \left( \frac{0.320}{0.218} \right) = 107 \frac{\text{m}}{\text{s}} \quad \text{Ans. (a)}
\]

\[
p_2 = 550 \left( \frac{3.682}{5.455} \right) = 371 \text{ kPa} \quad \text{Ans. (b)}
\]

\[
T_2 = 333 \left( \frac{1.179}{1.190} \right) = 330 \text{ K} \quad \text{Ans. (c)}
\]
Now we need $p_{o1}$ to get $p_{o2}$:

$$p_{o1} = 550[1 + 0.2(0.2)^2]^{3.5} \approx 566 \text{ kPa}, \quad \text{so} \quad p_{o2} = 566 \left( \frac{2.067}{2.964} \right) \approx 394 \text{ kPa}$$

The extra distance we need to choke the exit to sonic speed is $(fL/D)_2 = 5.53$. That is,

$$\Delta L = 5.53 \frac{D}{f} = 5.53 \left( \frac{0.03}{0.018} \right) \approx 9.2 \text{ m} \quad \text{Ans.}$$

9.87  Air enters an adiabatic duct of $L/D = 40$ at $V_1 = 170 \text{ m/s}$ and $T_1 = 300 \text{ K}$. The flow at the exit is choked. What is the average friction factor in the duct?

**Solution:** Noting that $Ma_{\text{exit}} = 1.0$, compute $Ma_1$, find $fL/D$ and hence $f$:

$$Ma_1 = \frac{V_1}{a_1} = \frac{170}{\sqrt{1.4(287)(300)}} = \frac{170}{347} = 0.49,$$

Table B.3: read $fL/D \approx 1.15$ Then $f = \frac{1.15}{40} \approx 0.029 \quad \text{Ans.}$

9.88  Air enters a 5- by 5-cm square duct at $V_1 = 900 \text{ m/s}$ and $T_1 = 300 \text{ K}$. The friction factor is 0.02. For what length duct will the flow exactly decelerate to $Ma = 1.0$? If the duct length is 2 m, will there be a normal shock in the duct? If so, at what Mach number will it occur?

**Solution:** First compute the inlet Mach number, which is decidedly supersonic:

$$Ma_1 = \frac{V_1}{a_1} = \frac{900}{\sqrt{1.4(287)(300)}} \approx 2.59,$$

read $(fL/D)_1 = 0.451$, whence $L^*|_{Ma=1} = 0.451 \left( \frac{0.05}{0.02} \right) = 1.13 \text{ m} \quad \text{Ans.}$

[We are taking the “hydraulic diameter” of the square duct to be 5 cm.] If the actual duct length = 2 m $> L^*$, then there must be a normal shock in the duct. By trial and error, we need a total dimensionless length $(fL/D) = 0.02(2)/0.05 \approx 0.8$. The result is:

$$Ma_1 = 2.59, \quad \frac{fL}{D}|_1 = 0.451, \quad Ma_2 = 2.14, \quad \frac{fL}{D}|_2 = 0.345,$$

shock: $Ma_3 = 0.555, \quad \frac{fL}{D}|_3 = 0.695$

Total $fL/D = 0.451 - 0.345 + 0.695 = 0.801 \text{ (close enough)} \quad \therefore \quad Ma_2 = 2.14 \quad \text{Ans.}$
9.90 Air, supplied at $p_0 = 700$ kPa and $T_0 = 330$ K, flows through a converging nozzle into a pipe of 2.5-cm diameter which exits to a near vacuum. If $f = 0.022$, what will be the mass flow through the pipe if its length is (a) 0 m, (b) 1 m, and (c) 10 m?

**Solution:** (a) With no pipe ($L = 0$), the mass-flow is simply the isentropic maximum:

$$
\dot{m} = \dot{m}_{\text{max}} = 0.6847 \frac{p_0 A^*}{\sqrt{RT_0}} = 0.6847 \frac{700000(\pi/4)(0.025)^2}{\sqrt{287(330)}} = 0.764 \text{ kg/s} \quad \text{Ans. (a)}
$$

(b) With a finite length $L = 1$ m, the flow will choke in the exit plane instead:

$$
Ma_e = 1.0, \quad \frac{fL}{D} = \frac{0.022(1.0)}{0.025} = 0.88, \quad \text{read } Ma_1(\text{entrance}) \approx 0.525
$$

Then

$$
T_1 = 330/[1 + 0.2(0.525)^2] = 313 \text{ K}, \quad a_1 = \sqrt{1.4(287)(313)} \approx 354 \text{ m/s},
$$

$$
V_1 = Ma_1a_1 = 186 \text{ m/s}, \quad p_1 = 700/[1 + 0.2(0.525)^2]^{3.5} = 580 \text{ kPa},
$$

$$
\rho_1 = p_1/(RT_1) = 6.46 \text{ kg/m}^3
$$

Finally, then,

$$
\dot{m} = \rho_1 A_1 V_1 = (6.46)(\pi/4)(0.025)^2 (186) \approx 0.590 \text{ kg/s} \quad (23\% \text{ less}) \quad \text{Ans. (b)}
$$

(c) Repeat part (b) for a much longer length, $L = 10$ m:

$$
\frac{fL}{D} = \frac{0.022(10)}{0.025} = 8.8, \quad Ma_1 = 0.246, \quad T_1 = 326 \text{ K}, \quad a_1 = 362 \text{ m/s}, \quad V_1 = 89 \text{ m/s},
$$

also,

$$
p_1 = 671 \text{ kPa}, \quad \rho_1 = 7.17 \text{ kg/m}^3, \quad \dot{m} = \rho_1 A_1 V_1 = 0.314 \text{ kg/s} \quad (59\% \text{ less}) \quad \text{Ans. (c)}
$$

9.91 Air flows steadily from a tank through the pipe in Fig. P9.91. There is a converging nozzle on the end. If the mass flow is 3 kg/s and the flow is choked, estimate (a) the Mach number at section 1; and (b) the pressure in the tank.

![Fig. P9.91](image-url)
**Solution:** For adiabatic flow, \( T^* = \text{constant} = T_0/1.2 = 373/1.2 = 311 \text{ K} \). The flow chokes in the small exit nozzle, \( D = 5 \text{ cm} \). Then we estimate \( M_{a2} \) from isentropic theory:

\[
\frac{A_2}{A^*} = \left( \frac{6 \text{ cm}}{5 \text{ cm}} \right)^2 = 1.44, \quad \text{read } M_{a2} \text{ (subsonic)} \approx 0.45, \text{ for which } \frac{fL/D}{2} \approx 1.52,
\]

\[
p_{2}/p^* \approx 2.388, \quad p_{o2}/p^*_o \approx 1.449, \quad \rho_{2}/\rho^*_2 \approx 2.070, \quad T_2/T^* = 1.153 \text{ or } T_2 \approx 359 \text{ K}
\]

Given \( \dot{m} = 3 \frac{\text{kg}}{\text{s}} = \rho_2 A_2 V_2 = \frac{p_2}{287(359)} \left( \frac{\pi}{4} \right)(0.06)^2(0.45)\sqrt{1.4(287)(359)}, \)

Solve for \( p_2 \approx 640 \text{ kPa} \). Then \( p^* = 640/2.388 \approx 268 \text{ kPa} \)

At section 1, \( \frac{fL}{D} = \frac{fL}{D}_{2} + \frac{f\Delta L}{D} = 1.52 + \frac{0.025(9)}{0.06} = 5.27, \text{ read } M_{a1} \approx 0.30 \text{ Ans. (a)} \)

for which \( p_1/p^* \approx 3.6, \text{ or } p_1 \approx 3.6(268) \approx 965 \text{ kPa} \).

Assuming isentropic flow in the inlet nozzle,

\[
p_{\text{tank}} = 965[1 + 0.2(0.30)^2]^{3.5} \approx 1030 \text{ kPa} \text{ Ans. (b)}
\]

**9.92** Modify Prob. 9.91 as follows: Let the tank pressure be 700 kPa, and let the nozzle be choked. Determine (a) \( M_{a2} \); and (b) the mass flow. Keep \( T_0 = 100^\circ \text{C} \).

**Solution:** This is the reverse of Prob. 9.91 and is easier. The Mach numbers are the same, since they depend only upon \( fL/D \) (which is the same) and the two nozzle area ratios. If we didn’t know the solution to Prob. 9.91, we would guess \( M_{a1} \), work out \( M_{a2} \) and see if the flow then expands exactly to a sonic exit at the second nozzle. Repeat, if necessary, until the progression through the pipe and the second nozzle is choked. The results are:

\( M_{a1} = 0.30, \text{ compute } p_1 = 700/[1 + 0.2(0.30)^2]^{3.5} \approx 658 \text{ kPa} \). In Table B.3, read

\( p_1/p^* \approx 3.6, \text{ or } p^* = \frac{658}{3.6} \approx 183 \text{ kPa} \). Also read \( fL/D \approx 5.27 \), subtract \( f\Delta L/D \) of 3.75 to find \( fL/D \approx 1.52 \), read \( M_{a2} \approx 0.45 \text{ Ans. (a)} \) Table B.1: \( A_2/A^* \approx 1.44 \)

Then \( A_{\text{exit}}/A^* = \frac{1.44}{(6/5)^2} \approx 1.0 \) (exactly what we want, sonic flow exit).

Go back to sections 1 or 2 to compute \( \dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = 2.04 \text{ kg/s} \text{ Ans. (b)} \)

**9.93** Air flows adiabatically in a 3-cm-diameter duct with \( f = 0.015 \). At the entrance, \( V = 950 \text{ m/s} \) and \( T = 250 \text{ K} \). How far down the tube will (a) the Mach number be 1.8; and (b) the flow be choked?
A heat addition of 504 kJ/kg is (just barely) less than maximum, should nearly choke:

\[
T_{o2} = T_{o1} + \frac{q}{c_p} = 305 + \frac{540000}{1005} \approx 842 \text{ K}, \quad \frac{T_{o2}}{T_o^*} = \frac{842}{880} = 0.957, \quad \therefore \text{Ma}_2 \approx 0.78 \quad \text{Ans. (b)}
\]

Finally, without using Table B.4, \[T_2 = 842 / \left[1 + 0.2(0.78)^2\right] \approx 751 \text{ K} \quad \text{Ans. (c)}
\]

9.108 What happens to the inlet flow of Prob. 9.107 if the combustion yields 1500 kJ/kg heat addition and \(p_{o1}\) and \(T_{o1}\) remain the same? How much is the mass flow reduced?

Solution: The flow will choke down to a lower mass flow such that \(T_{o2} = T_o^*\):

\[
T_{o2} = T_o^* = 305 + \frac{1500000}{1005} = 1798 \text{ K}, \quad \text{thus} \quad \frac{T_{o1}}{T_o^*} = \frac{305}{1798} = 0.17, \quad \text{Ma}_{1,\text{new}} \approx 0.198
\]

\[(\dot{m}/A)_{\text{new}} = \rho_1 V_1 = \rho_0 \text{Ma}_1 = \rho_0 \text{a}_0 \text{Ma}_1 / [1 + 0.2(\text{Ma}_1^2)]^3 \quad \text{if} \quad p_{o1}, \ T_{o1}, \ \rho_{o1} \ \text{are the same.}
\]

Then \[
\frac{\dot{m}_{\text{new}}}{\dot{m}_{\text{old}}} = \frac{0.198}{0.30} \left[\frac{1 + 0.2(0.30)^2}{1 + 0.2(0.198)^2}\right] \approx 0.68 \quad \text{about 32% less flow} \quad \text{Ans.}
\]

9.109 A jet engine at 7000-m altitude takes in 45 kg/s of air and adds 550 kJ/kg in the combustion chamber. The chamber cross section is 0.5 m², and the air enters the chamber at 80 kPa and 5°C. After combustion the air expands through an isentropic converging nozzle to exit at atmospheric pressure. Estimate (a) the nozzle throat diameter, (b) the nozzle exit velocity, and (c) the thrust produced by the engine.

\[
\text{Fig. P9.109}
\]

Solution: At 700-m altitude, \(p_a = 41043 \text{ Pa}, \ T_a = 242.66 \text{ K}\) to use as exit conditions.

\[
\rho_1 = \frac{p_1}{RT_1} = \frac{80000}{287(278)} = 1.00 \quad \text{kg/m}^3, \quad \dot{m} = 45 \quad \text{kg/s} = \rho AV = 1.00(0.5)V_1, \quad V_1 = 90 \text{ m/s}
\]
Chapter 9 • Compressible Flow

Ma₁ = \frac{90}{\sqrt{1.4(287)(278)}} = 0.27, \quad \text{Table B.4: } T₀₁/T₀^* ≈ 0.29,

T₀₁ = 278 + (90)^2/[2(1005)] ≈ 282 K, \quad \therefore T₀^* = 282/0.29 ≈ 973 K

Add heat: T₀₂ = 282 + \frac{550000}{1005} = 829 K, \quad \text{thus } \frac{T₀₂}{T₀^*} = \frac{829}{973} = 0.85, \quad \text{read } Ma₂ = 0.63

also read \quad p₁/p^* = 2.18, \quad p₂/p^* = 1.54, \quad \therefore \quad p₂ = 80(1.54/2.18) ≈ 57 kPa,

p₀₂ = p₂ \left[1 + 0.2Ma₂^{3.5}\right] = 57[1 + 0.2(0.63)^2]^{3.5} \approx 74 kPa

With data now known at section 2, expand isentropically to the atmosphere:

\frac{Pₑ}{P₀₂} = \frac{41043}{57000} = 0.72 = \left[1 + 0.2Maₑ^{3.5}\right], \quad \text{solve } Maₑ = 0.70, \quad \frac{Tₑ}{T₀₂} = \frac{Tₑ}{829} = 0.910 ,

Solve \quad Tₑ \approx 755 K, \quad \rhoₑ = \frac{Pₑ}{RTₑ} \approx 0.189 \text{ kg/m}³, \quad aₑ = \sqrt{kRTₑ} = 551 \text{ m/s}, \quad Vₑ = Maₑaₑ = 385 \text{ m/s} \quad \text{Ans. (b)}

\dot{m} = 45 = 0.189(385)\pi Dₑ², \quad \text{solve } Dₑ = 0.89 \text{ m} \quad \text{Ans. (a)}

Finally, if pₑ = pₐₜₘ, \quad \text{from Prob. 3.68, } Fₜₜₘₐᵋₜ = \dot{m}Vₑ = 45(385) = 17300 N \quad \text{Ans. (c)}

9.110 Compressible pipe flow with heat addition, Sec. 9.8, assumes constant momentum (p + ρV²) and constant mass flow but variable stagnation enthalpy. Such a flow is often called Rayleigh flow, and a line representing all possible property changes on an temperature-entropy chart is called a Rayleigh line. Assuming air passing through the flow state p₁ = 548 kPa, T₁ = 588 K, V₁ = 266 m/s, and A = 1 m², draw a Rayleigh curve of the flow for a range of velocities from very low (Ma ≪ 1) to very high (Ma ≫ 1). Comment on the meaning of the maximum-entropy point on this curve.

Solution: \quad \text{First evaluate the Mach number and density at the reference state:}

\rho = \frac{p}{RT} = \frac{548000}{287(588)} \approx 3.25 \frac{\text{kg}}{\text{m}³}, \quad \text{Ma} = \frac{V}{\sqrt{kRT}} = \frac{266}{\sqrt{1.4(287)(588)}} = 0.55

Our basic algebraic equations are then:

Momentum: \quad p + \rho V² = 548000 + 3.25(266)² = 778000 \quad \text{(a)}

Continuity: \quad \rho V = 3.25(266), \quad \text{or: } \rho = 864/V \quad \text{(b)}

Entropy: \quad s = 718\ln(T/588) - 287\ln(\rho/3.25) \quad \text{(c)}
Then \( T_{o1}/T_0^* = \frac{301}{840} = 0.358 \); Table B.4: read \( Ma_1 = 0.306 \), read \( V_1/V^* \approx 0.199 \)

So \( V_1 = 530(0.199) = 105 \text{ m/s} \quad \text{Ans. (a)} \)

Also read \( p_{o1}/p_o^* \approx 1.196 \), where \( p_o^* = p_o/0.5283 \approx 180 \text{ kPa} \),

Hence \( p_{o1} \approx 1.196(180) \approx 215 \text{ kPa} \quad \text{Ans. (b)} \)

9.113 Air enters a constant-area duct at \( p_1 = 90 \text{ kPa}, V_1 = 520 \text{ m/s}, \text{ and } T_1 = 558^\circ\text{C}. \) It is then cooled with negligible friction until it exists at \( p_2 = 160 \text{ kPa}. \) Estimate (a) \( V_2 \); (b) \( T_2 \); and (c) the total amount of cooling in \( \text{kJ/kg} \).

**Solution:** We have enough information to estimate the inlet \( Ma_1 \) and go from there:

\[
a_1 = \sqrt{1.4(287)(558 + 273)} = 578 \text{ m/s}, \quad \therefore \quad \text{Ma}_1 = \frac{520}{578} \approx 0.90, \quad \text{read } \frac{p_1}{p^*} = 1.1246,
\]

or \( p^* = \frac{90}{1.1246} = 80.0 \text{ kPa} \), whence \( \frac{p_2}{p^*} = \frac{160}{80} \approx 2.00, \quad \text{read } \text{Ma}_2 \approx 0.38, \)

\[
\text{read } \frac{T_2}{T^*} = 0.575, \quad \frac{V_2}{V^*} = 0.287, \quad \frac{T_02}{T_0^*} = 0.493
\]

We have to back off to section 1 to determine the critical (*) values of \( T, V, T_0 \):

\[
\text{Ma}_1 = 0.9, \quad \frac{T_1}{T^*} = 1.0245, \quad T^* = \frac{558 + 273}{1.0245} = 811 \text{ K}, \quad T_2 = 0.575(811) \approx 466 \text{ K} \quad \text{Ans. (b)}
\]

also, \( \frac{V_1}{V^*} = 0.911, \quad V^* = \frac{520}{0.911} = 571 \text{ m/s}, \quad \text{so } V_2 = 0.287(571) \approx 164 \text{ m/s} \quad \text{Ans. (a)}
\]

\[
\frac{T_{o1}}{T_o^*} = 0.9921, \quad \text{where } T_{o1} = T_1 + \frac{V_1^2}{2c_p} = 966 \text{ K}, \quad T_o^* = \frac{966}{0.9921} = 973 \text{ K}
\]

Finally, \( T_{o2} = 0.493(973) = 480 \text{ K}, \)

\[
\text{q}_{\text{cooling}} = c_p\Delta T_o = 1.005(966 - 480) \approx 489 \text{ kJ/kg} \quad \text{Ans. (c)}
\]

9.114 We have simplified things here by separating friction (Sec. 9.7) from heat addition (Sec. 9.8). Actually, they often occur together, and their effects must be evaluated simultaneously. Show that, for flow with friction and heat transfer in a constant-diameter pipe, the continuity, momentum, and energy equations may be combined into the following differential equation for Mach-number changes:

\[
\frac{dMa^2}{Ma^2} = \frac{1 + kMa^2}{1 - Ma^2} \frac{dQ}{c_pT} + \frac{kMa^2[2 + (k-1)Ma^2]}{2(1-Ma^2)} f \frac{dx}{D}
\]