EE520, Bollt,

Proj-HW 1, SVD, PCA, POD, KL, due W Sept 18

To be presented in a professional report style format, including question statements. Answers in a narrative technical format. Figures, with figure captions as appropriate describing what is to be observed. Figures referred to in text. Tables, likewise. Codes to be also presented.

1. Image of a circle. (Matlab). Make a drawing of a circle of radius r. Plot the image of a unit circle in R^2 when each point is multiplied by A = (3 -2; -1 5). Also overlay the scaled left singular vectors σ1u1 and σ2u2 on your plot and verify that they line up with the axes of the ellipse. Does the placement of the (center) of the circle matter? Discuss.

2. Very simple linear regression. Consider the three points (0,1), (1,-1), (2,-2). What is the equation of the line that best fits those three points, in the sense of least squares?

3. Reduced order modeling with respect to the Euclidean norm, is a big deal, and ubiquitious as an application of SVD. (Just say **YES**!)

4. Dimension reduction.

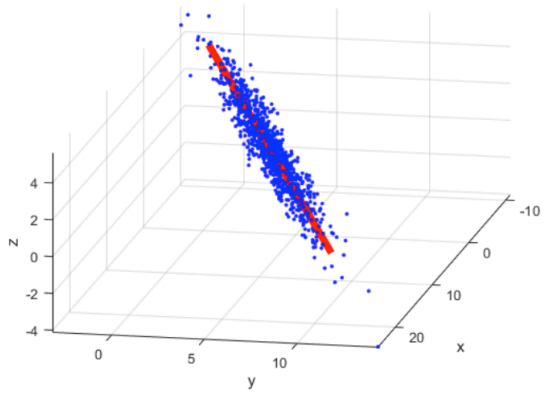
Load the file sdata.csv which contains a 1000 × 3 matrix of data. Each row of the matrix is a point (x_i, y_i, z_i) in R^3. We will approximate this data set as an affine one- dimensional space (a line that doesn't pass through the origin).
a) Find the line that best approximates the data in the sense of minimizing the sum of the squares of the projections of all points onto the line. Plot the line and the data on the same axes and verify that the line approximates the points. Hint: before finding the line, shift every point so that the data has zero mean. You can make 3D scatter plots in Matlab by using plot3
b) Instead of using three numbers (x_i, y_i, z_i) to describe each data point, we can now use a singlenumber w_i, which is the position along the line of the projected data point. Give a formula that
converts (x,y,z) to w and the reverse formula, which converts w to a point (x,y,z).
c) Convert the data set to w_i
coordinates, and plot a histogram of the {w_i} to see how the points
are distributed. Use 20 equally spaced bins for the histogram.

Big Hint:

```
A = csvread('sdata.csv')
xm = mean(A,1);                          % compute mean of all the points
[u,s,v] = svd(A - repmat(xm,N,1));       % subtract mean from each row
v1 = v(:,1);                             % leading right singular vector
s1 = s(1,1);                             % largest singular value

figure(1)
plot3(A(:,1),A(:,2),A(:,3),'b.')
xlabel('x'); ylabel('y'); zlabel('z')
axis equal; grid on

% plot vector v1 (shifted by the mean) together with the data
t = linspace(-1,1);
hold on; plot3( xm(1)+t*s1*v1(1), xm(2)+t*s1*v1(2), ...
                xm(3)+t*s1*v1(3), 'r-','LineWidth',5 )
```



5. Compute the SVD in closed form, by hand, of the matrix, A=[1 -1; 0 1; 1 0];

6.

For a matrix G, let $\{u_1, \ldots u_m\}$ be the left singular vectors, let $\{v_1, \ldots, v_n\}$ be the right singular vectors and let $\sigma_1, \ldots, \sigma_r$ be the non-zero singular values. Starting from the SVD of G show that

$$G = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_r u_r v_r^T.$$

7.

Let us consider the matrix A and its singular value decomposition given below.

$$A = \begin{bmatrix} 11 & 8 & 5 & 8 \\ -10 & -12 & -14 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{13} & 3/\sqrt{13} \\ -3/\sqrt{13} & 2/\sqrt{13} \end{bmatrix} \begin{bmatrix} 8\sqrt{13} & 0 & 0 & 0 \\ 0 & 2\sqrt{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 0 & 1/2 & 0 & -1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{bmatrix}. \quad (1)$$

(a) Write down the singular values, and corresponding left and right singular vectors for A.

(b) Express A in orthogonal rank 1 form, that is express A as a sum of outer products that are mutually orthogonal.

find an orthonormal basis for the null space of A;

find a matrix $B \in \mathbb{R}^{2\times 4}$ of rank one, and is such that $\|B - A\|_2$ is as small as possible.

8. Compress your professor. Reduce order professor. There is an image of a *younger* professor giving a lecture locally at google images.
http://www.soarnorthcountry.com/images/g_image/photo-007413.jpg

Download it and save it in an appropriate place. In Matlab you can

open this image by typing
H = imread('whateveryoucalledit.jpg');

imshow(H) allows you to look at it from Matlab

9. Eigenfaces. Perform an eigenface analysis of the data set provided. Here is an example of the kind of questions such an analysis would include, but this question is mostly open ended for you to "discuss" and code yourself.

10. Show me either a photograph, or a screenshot, of you meeting with at least 2 other students from at least 2 occasions, but the work you hand in will be your own "hand". (By this, I am encouraging you to collaborate).