Controlling Chaotic Motions of a Nonlinear Aeroelastic System Using Adaptive Control Augmented with Time Delay

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ABSTRACT

In this paper the application of an adaptive reference-free feedback controller to a nonlinear aeroelastic system subjected to steady aerodynamic loads is examined. The numerical simulations show that when using the angle-of-attack as a feedback signal, the system chaotic motion and LCOs can be suppressed by properly selecting the adaptive filter cut-off frequency and the feedback gain. In addition the controller is augmented with a time delay parameter; it has been determined that it is possible to obtain the regions of instability due to time delay in a nonlinear aeroelastic system under adaptive closed loop control.

INTRODUCTION

Aeroelasticity has been and continues to be an important consideration in the design of primary flight structures. In particular, dynamic aeroelastic effects such as flutter, limit cycle oscillations and in some instances chaotic motion can place severe operational constraints on flight vehicle performance. The presence of limit cycles in aircraft structures and rotor blades has made it necessary for aeroservoelasticians to investigate the feasibility of adaptive control systems (both linear and nonlinear) to address LCOs. From an airworthiness perspective, LCOs can result in fatigue as in the case of turbomachinery or twined tailed fighter aircraft and in some instances lead to catastrophic failure. As a result it is important to understand how to transition and confine an aeroelastic system to a limit cycle that is entrained in chaotic motion and suppress the limit cycle altogether.

Before we commence the discussion on limit cycle control and suppression, it is instructive to understand the mechanism by which limit cycles and chaotic motions are created. A limit cycle is a standing periodic oscillation that is characterized in the phase plane as a single loop; in some instances a multi-frequency limit cycle may appear as in the case of a chaotic system that has been transformed by an appropriate control force. For a stable limit cycle, the rate of energy input from the freestream is equal to the energy dissipation rate. A limit cycle that is unstable under the following physical conditions; when the system continuously receives more energy than it is able to dissipate, the limit cycle will grow in amplitude. Conversely, if energy is extracted from the system, then the oscillation will decay. It is this mechanism that makes it possible to force an aeroelastic system to switch limit cycles. We will revisit the notion of an

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unstable limit cycle and limit cycle switching later. For now we will introduce examples of limit cycle switching and suppression as it applies to aeroservoelasticity.

The first such example as pointed out by Dimitriadias and Cooper [1] was work performed by Holden et al [2]. Specifically, during a series of wind tunnel tests on a flutter model of a tail plane, Holden and his colleagues noticed that applying a certain excitation signal caused an LCO; the re-application of the same signal a short time later produced a limit cycle of larger amplitude. The manipulation of limit cycles has a lot to do with the energy state of the system. There are devices which have the capability to make instantaneous changes of its mass, stiffness or damping; such devices are termed state switchable dynamical systems. For example if a switchable stiffness element is build into a vibration absorber, the change in stiffness causes a change in the resonant frequencies of the system and thus 'retuning' the system. One can design a switching rule-control law that extracts energy from the system [3,4]. Lee et al [5] demonstrated that one can use continuously varying stiffness and damping elements (or nonlinear energy sinks –NES) coupled to a van der Pol oscillator. By adjusting the parameters of the NES indicated in Figure 1, suppression of LCOs can be achieved.

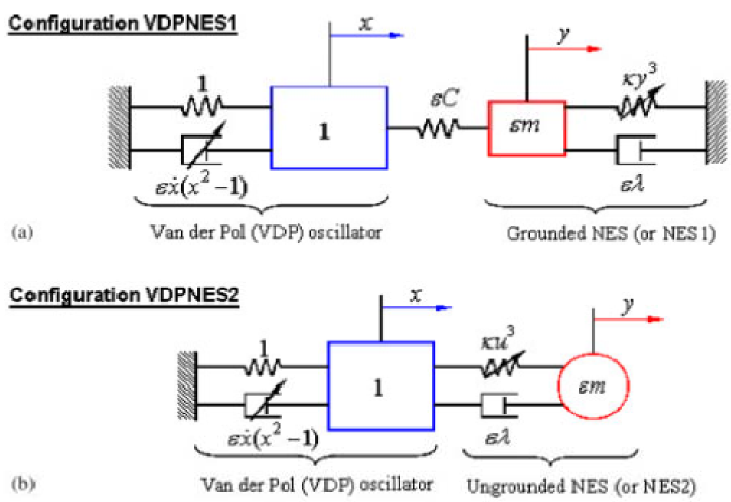

Figure 1. System configuration with: (a) grounded NES; (b) ungrounded NES [5]

Now let's revisit the discussion on unstable limit cycles also referred to as unstable periodic orbits (UPOs); for a system with sufficient complexity (i.e. multiple degrees of freedom, number and type of nonlinearities), a significant number of limit cycles can exist in its phase plane. As such the key is to force a system into a stable limit cycle knowing the location of the unstable limit cycles that surround it. A very similar argument was made by Pyragas [6]. Specifically; the stabilization of unstable periodic orbits of a chaotic system is achieved by applying a combined feedback with the use of a specially designed external oscillator or by a delayed self controlling (Figure 2) feedback force without the use of an external force. Both of these methods do not require any prior analytical knowledge of the system. These methods make use of the fact that there are an infinite number of unstable periodic orbits contained in a chaotic attractor. The delayed-self controlling method was successfully demonstrated for an aeroelastic system by Ramesh et al [7].

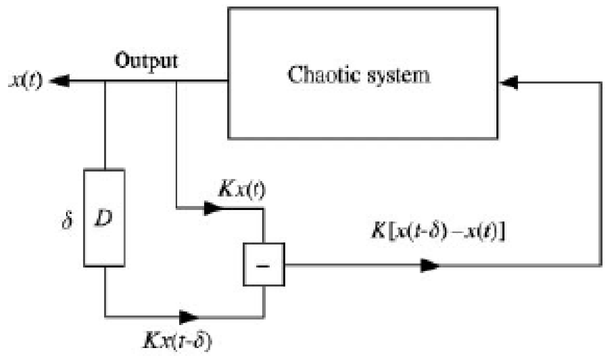

Figure 2. Schematic of delayed-feedback control system [6].

However suppression is a necessary requirement for airworthiness and certification. To achieve full suppression of the said chaotic system, it is the objective of this paper to show that the application of a simple adaptive controller which is based on a low pass filter is able to stabilize a highly dynamic aeroelastic system. In addition to the LCO suppression study, we will also address the effect of time delay on an adaptive flutter suppression algorithm. Short time delays in control systems are unavoidable especially when digital controller are used. Other sources of time delay are actuators, sensors, mechanical linkages, and filters. [8]. Time delays can significantly impact a closed-loop system if the control demand induces large control forces or if the controller is unable to handle high frequencies. Time delays when appropriately applied can constrain an aeroelastic system to a limit cycle as demonstrated by Ramesh et al [7]. The application of or occurrence of a time delay event at the wrong time can degrade the control system performance [9]. It is important for aerospace engineers to understand the effect of inherent time delays so that they can be mitigated or designed out early in the flight dynamics work up on a major aircraft program. The effect of time delays have been studied for a simple aeroelastic system in [8,9]. In particular, the effect on the flutter boundary has been covered in the literature with full-state feedback control; see e.g. [12,13,14]. The present paper only addresses the lifting surface post-flutter behavior and its control through the adaptive reference-free feedback controller, while the flutter problem will be addressed elsewhere.

AEROELASTIC MODELING

The model used for this work is a two degree of freedom aeroelastic model shown below in Figure 3.

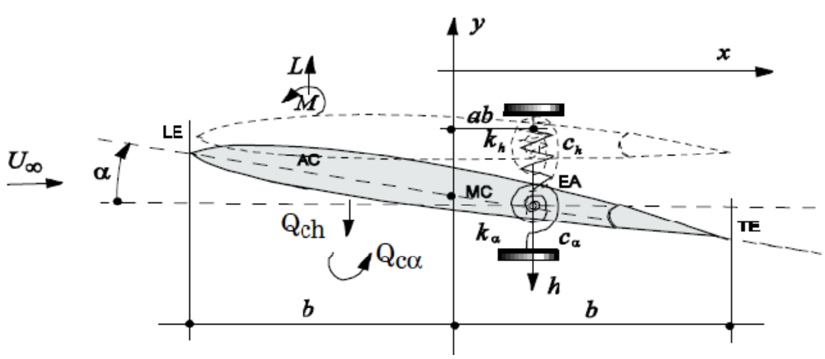

Figure 3. 2DOF Aeroelastic Model

The classical equations that describe the system are given below:

$$\mu\ddot{h} + \mu\chi_\alpha \ddot{\alpha} + \zeta_h \dot{h} + \mu\left(\frac{\omega_h}{\omega_\alpha}\right)^2 h = -\frac{Q_h}{\pi\rho b^3\omega_\alpha^2} \tag{1}$$

$$\mu\chi_\alpha\ddot{h} + \mu r_\alpha^2\ddot{\alpha} + \zeta_\alpha \dot{\alpha} + \mu r_\alpha^2\alpha + \frac{\varepsilon}{\pi\rho b^4\omega_\alpha^2}\alpha^3 = \frac{Q_\alpha}{\pi\rho b^4\omega_\alpha^2} + F(\tau) \tag{2}$$

where h and α are the bending and angular (pitch) displacements, $\mu = m/4\rho_\infty b^2$ is the mass ratio $\chi_\alpha = S_\alpha/mb$ is the dimensionless static unbalance about the elastic axis, EA.; b is the half chord length; $\zeta_h = c_h/(\pi\rho_\infty b^4\omega_\alpha)$, $\zeta_\alpha = c_\alpha/(\pi\rho_\infty b^4\omega_\alpha)$ are the dimensionless damping coefficients for bending and pitch respectively; ω_h and ω_α are the uncoupled frequencies of the aeroelastic system in bending and pitch respectively; r_α is the radius of gyration; Q_h and Q_α are the steady aerodynamic force and moment acting on the wing structure. $\tau = tU_\infty/b$ is the nondimensional time and $F(\tau)$ is the control force. A nonlinear structural hardening contribution is included as a cubic stiffness term in Eq. (2). This is a classical way of including a continuous nonlinear restoring moment, examples are provided in [7,10]. Numerical simulations are performed using the aeroelastic governing equations based on parameters provided in [9] and adapted to give the following:

$$\ddot{h} + 0.25\ddot{\alpha} + 0.1\dot{h} + 0.05h + (0.1Q)\alpha = 0 \tag{3}$$

$$\ddot{\alpha} + 0.5\ddot{h} + 0.2\dot{\alpha} + 2(0.5 - 0.04Q)\alpha + 40\alpha^3 = F(\tau) \tag{4}$$

The given system equations have the flutter speed is $Q_F = 6$. Figure 4 shows the airfoil in the open loop post flutter condition at Q=16.8.

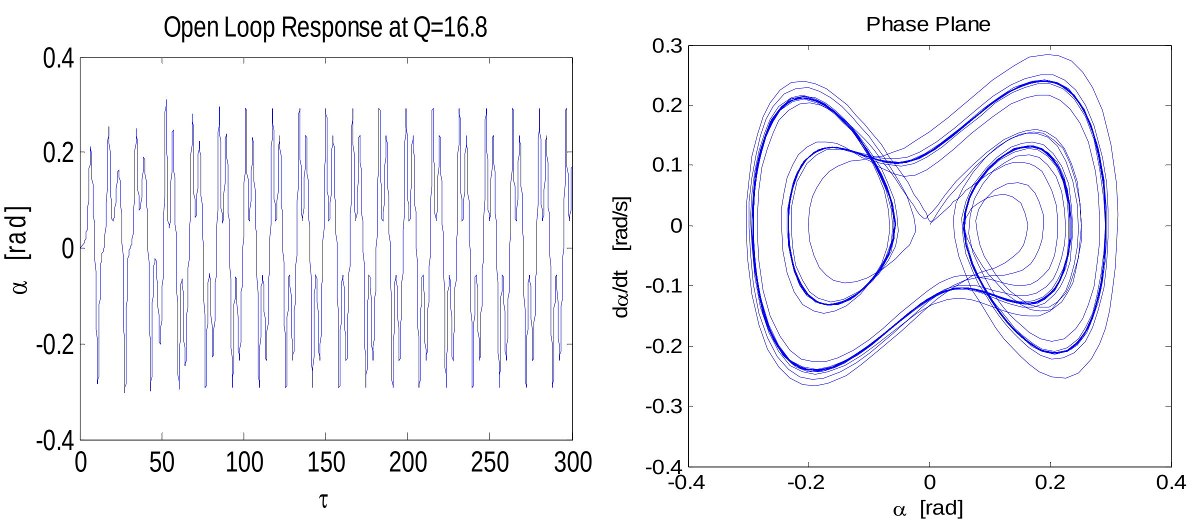

Figure 4. Post flutter open loop regime

ADAPTIVE CONTROLLER

Controlling chaotic behavior mainly deals with the stabilization of unstable periodic orbits. The stabilization of a fixed point by classical methods requires knowledge of the location in the phase space. For many complex systems, the locations of the fixed points are not known a priori; as such, adaptive control techniques that are capable of locating unknown steady states are desirable. A simple adaptive controller for stabilizing unknown steady states can be designed using ordinary differential equations (ODEs). One such controller utilizes a first order ODE that represents a low pass filter (LPF). The filtered DC output of the filter estimates the location of the fixed point, such that the difference between the actual and the filtered signal can be used as a control signal. This control signal is then scaled by a proportional gain. The structure of the system is represented in Figure 5.

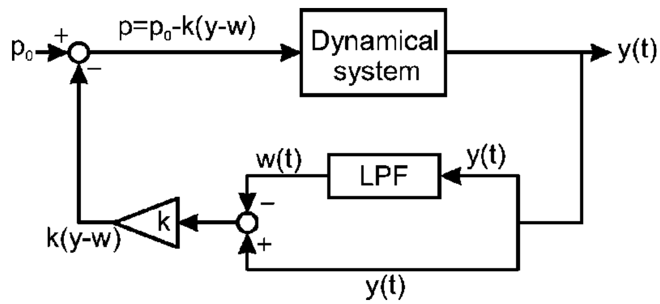

Figure 5. Block diagram of adaptive controller. LPF denotes low pass filter [11]

In general mathematical terms, when considering an autonomous dynamical system as the one in Figure 5, a description by a set of ordinary differential equations can be cast as:

$$\dot{x} = f(x, p) \tag{5}$$

where the vector $x \in R^m$ defines the dynamical variables and p is the total control force. The scalar variable $y(\tau) = g(x(\tau))$ is a function of the systems states; for the aeroelastic system, $y(\tau)$ could be anyone of the four states i.e. pitch, plunge, or any of their rates. Suppose that $p = p_0$, that is the system has an unstable fixed point at x^* that satisfies $f(x^*, p_0) = 0$. If the steady state value $y^*=g(x^*)$ corresponding to the fixed point were known, one could try to stabilize it using classical proportional feedback control.

$$p = p_0 - k(y - y^*) \tag{6}$$

Suppose now, that the reference value y^* is unknown. The objective will be to construct a reference-free feedback perturbation that automatically locates and stabilizes the fixed point.

When the controller locates the fixed point, the control input should vanish i.e. no control power should be dissipated in the closed-loop condition. The controller that satisfies the requirements can be constructed from an ODE that represents a low pass filter given by the following equation:

$$\dot{w} = \omega^c(y - w) \tag{7}$$

Here, w is a controller variable and the parameter, ω^c represents the cut-off frequency of the filter. The output of the filter provides an averaged input variable $y(\tau)$. If $y(\tau)$ oscillates about the steady state value of y^* one can expect that the output variable $w(\tau)$ will converge to this value. As result the reference value y^* can be replaced with the output variable of the filter; that is the control force can now be expressed in the following way:

$$p = p_0 - k(y - w) \tag{8}$$

The complete control-loop is in fact a high-pass filter, since the second term in Eq. (8) is obtained from the difference of the actual output signal and filtered by the LPF. The control signal is proportional to the derivative of the controller variable w i.e.

$$k(y - w) = \left(\frac{k}{\omega^c}\right)\dot{w} \tag{9}$$

As $\omega^c \rightarrow \infty$, from Eq. (5), it follows that $w(\tau) \rightarrow y(\tau)$; for large ω^c, the control signal becomes proportional to the derivative of the output $\dot{y}$. When this happens, the controller behaves as a simple derivative controller. In order to implement the time delay parameter, the output of the low pass filter, $w(t)$, is shifted by δ such that Eq. (8) can written in the following manner:

$$p(\tau) = p_0 - k\big(y(\tau) - w(\tau - \delta)\big) \tag{10}$$

For our numerical study, p_0 is set to zero so that the system is reference free and $p(\tau) \rightarrow F(\tau)$.

LIMIT CYCLE CONTROL AND SUPPRESSION

In this section, we present results on limit cycle control. The results consist of a parametric study and time history plots that pertain to the parametric system. Limit cycle control can be thought of as the act of extracting energy from the system such that the amplitude of the limit cycle decreases. For the aeroelastic system under investigation it is the control of its dynamics in the post flutter regime. Thus limit cycle control in this application means transforming the chaotic motion of the wing into a limit cycle. Specifically, a parametric study is used to gain insight into how the controller parameters affect the closed-loop system as well as provide guidance on which state variable to employ as a feedback signal. The feedback gain k, is evaluated at three filter cut-off frequencies; 0.001 rad/s, 0.01 rad/s and 0.1 rad/s.

Feedback Signal: Angular Displacement (α)

For a cut-off frequency ω^c=0.1 rad/s, the LCO amplitude decreases to 0.0941 rads (5.93 degrees) with a critical feedback gain of 1.6 Nm/rad (Figure 6) and remain stable to this amplitude for any higher feedback gain. A similar trend is observed for the cut-off frequency of 0.01 rad/s with a critical feedback gain of 1.4 Nm/rad. This is expected, since from theory, the properties of the controller are improved by decreasing the cut-off frequency ω^c [11]. Smaller values of ω^c are likely to stabilize more unstable foci. Stabilization in the context of this paper means full suppression. From the first two results it can be seen that stabilization has not been achieved. In fact the controller is dissipating control power in the closed-loop condition in order to maintain the LCO. The reason why lower cut-off frequencies work well can be explained by the interaction of the filter with the feedback signal; typically, aeroelastic systems can be characterized by their multi-harmonic limit cycles. We know from the section on the adaptive controller that the complete control-loop is a high pass filter; because the dynamics of the aeroelastic system exhibit multi-harmonics in the post flutter regime, only the first mode can be attenuated. So hence at higher cut-off frequencies, the filter is not able to attenuate the first mode and consequently the controller becomes saturated. Controller saturation in this context means that for large values of feedback gain combined with higher cut-off frequencies, the controller is unable to drive the state, in this case the angle-of-attack, to zero, thus this state remains at 0.0941 rads (for k= 5 Nm/rad) for these two filter settings.

For ω^c =0.001 rad/s, we observe that as the feedback gain increases, the amplitude decreases to zero at k=1.7 Nm/rad. From this result, it can be concluded that for low values of filter frequency combined with angular displacement as the feedback signal, results in limit cycle control and full suppression.

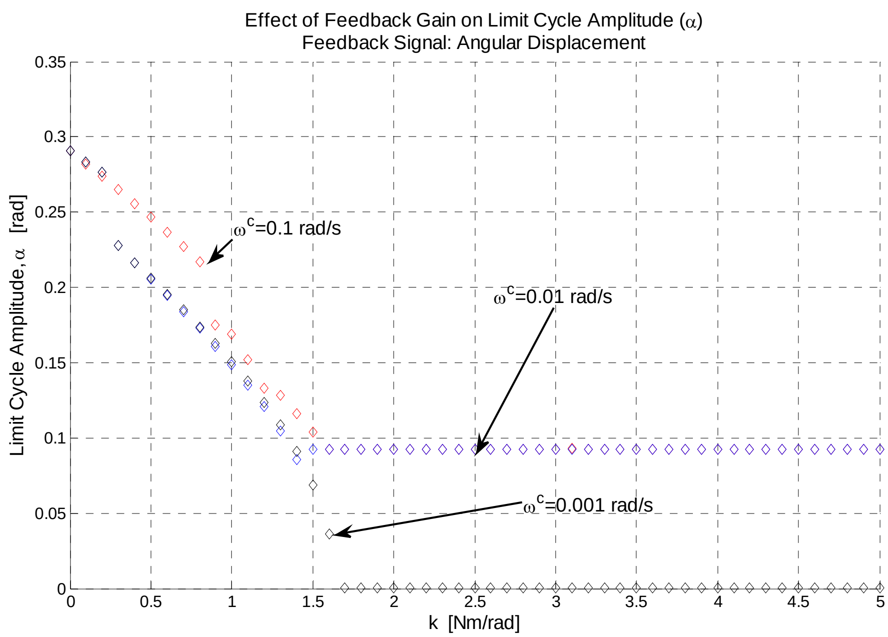

Figure 6. Effect of Feedback Gain on Limit Cycle Amplitude

Feedback Signal: Angular Velocity ($\dot{\alpha}$)

In the last section, the notion of a saturated controller was discussed. The same concept applies here as well. The change in amplitude as a function of feedback gain with the pitch rate as the feedback signal is compared to the previous result using the angular displacement as the feedback signal with the cut-off frequency at 0.001 rad/s (Figure 7). From the pitch rate parameter trajectory it can be seen that there are 2 inflection points, one at k=6.4 Nm s/rad and another at k=7 Nm s/rad. The reader will note that the using the pitch rate as a feedback signal requires the gain to be significantly higher than what is needed when angular displacement is used as the feedback signal.

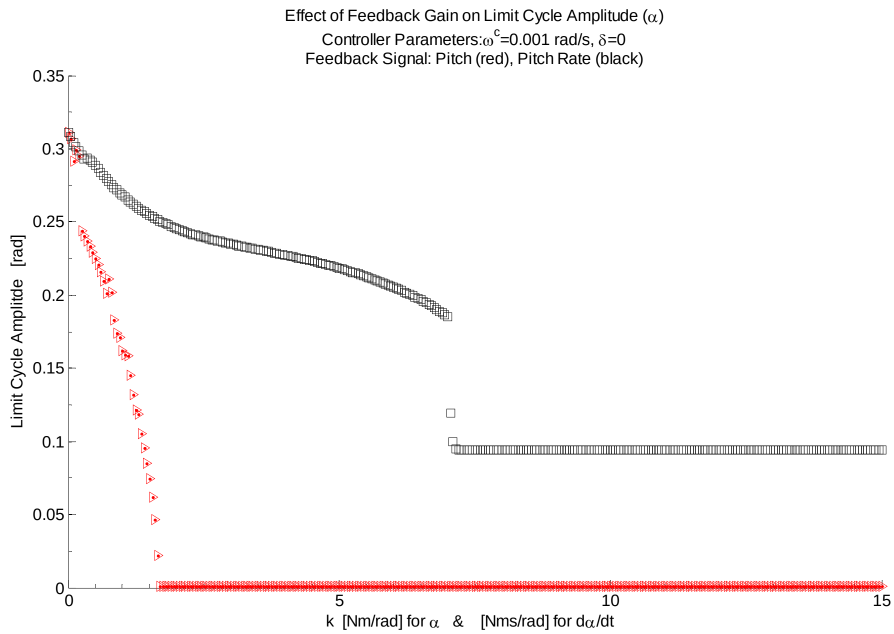

Figure 7. Effect of Feedback Gain on Limit Cycle Amplitude: Comparison

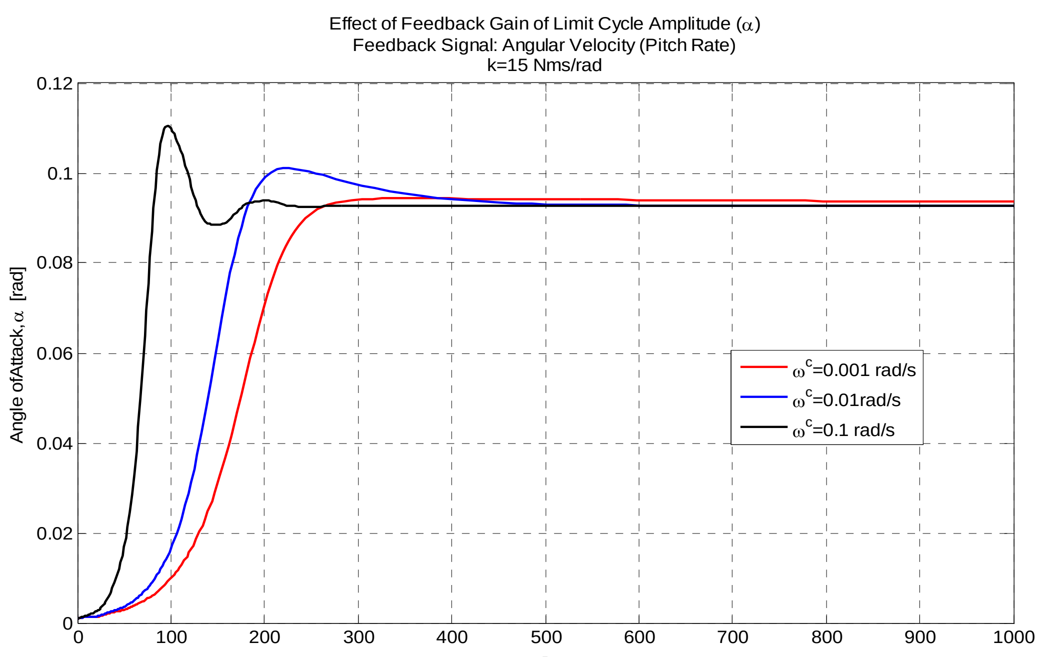

Figure 8. Effect of Cutoff Frequency at high gain

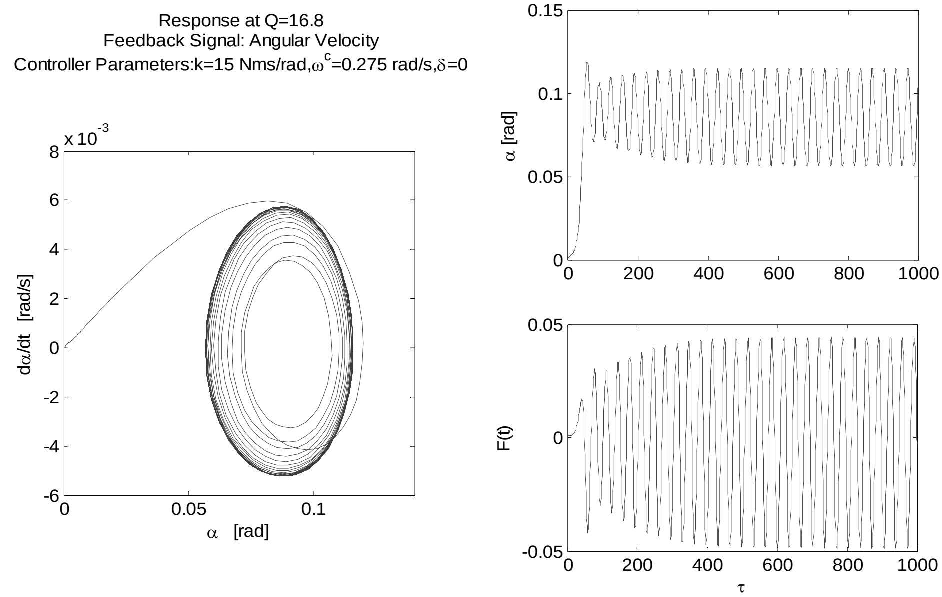

Figure 9. LCO with controller at high cutoff frequency and gain

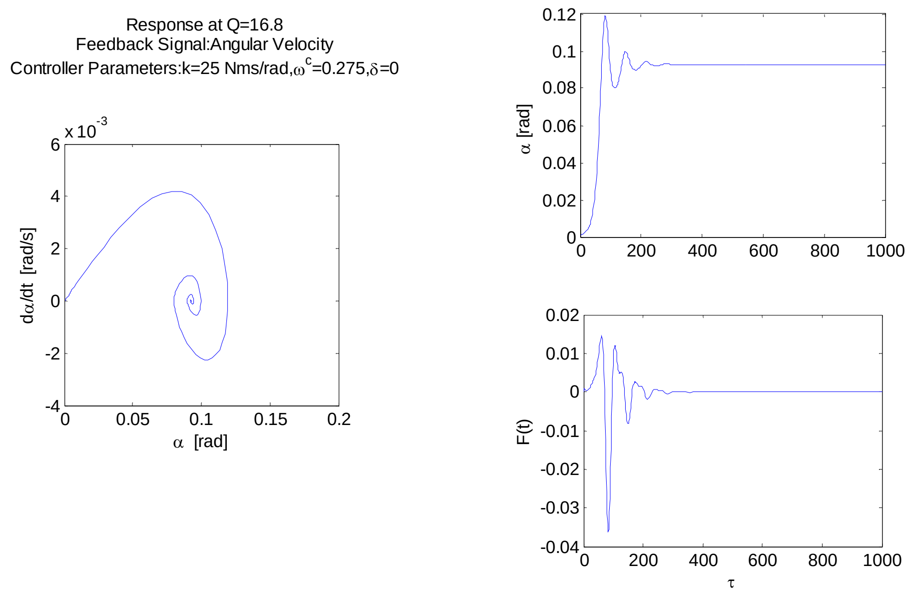

Figure 10. Full limit cycle suppression at high gain and cutoff frequency

Furthermore, when k is greater than 7 Nm s/rad the controller is not able to drive the angle-of-attack to zero but instead settles at a steady-state value of 0.0944 rads. The closed-loop system is however, very sensitive to changes in filter cut-off frequencies (Figure 8). Note the overshoot as the cut-off frequency increases. As the filter frequency increases, the closed-loop system will remain constrained in a limit cycle at k= 15 Nm s/rad (Figure 9); and as such it will require the feedback gain to be increased in order to suppress the limit cycle (Figure 10). Figure 11 shows the control case, when the control is turned on at τ = 0, in which the feedback gain is less than the critical value, k_{crit}= 1.7 Nm/rad for an ω^c=0.001 rad/s with the pitch displacement as the feedback signal (See Figure 6). Although the system is confined to a limit cycle, the control force amplitude remains constant. Figure 12, demonstrates that the controller is able to transform the post flutter chaotic dynamics into a stable limit cycle.In this particular simulation the controller is switched on at τ = 150. We see that the controller is able to suppress the chaotic dynamics and rapidly settle onto a limit cycle oscillation.

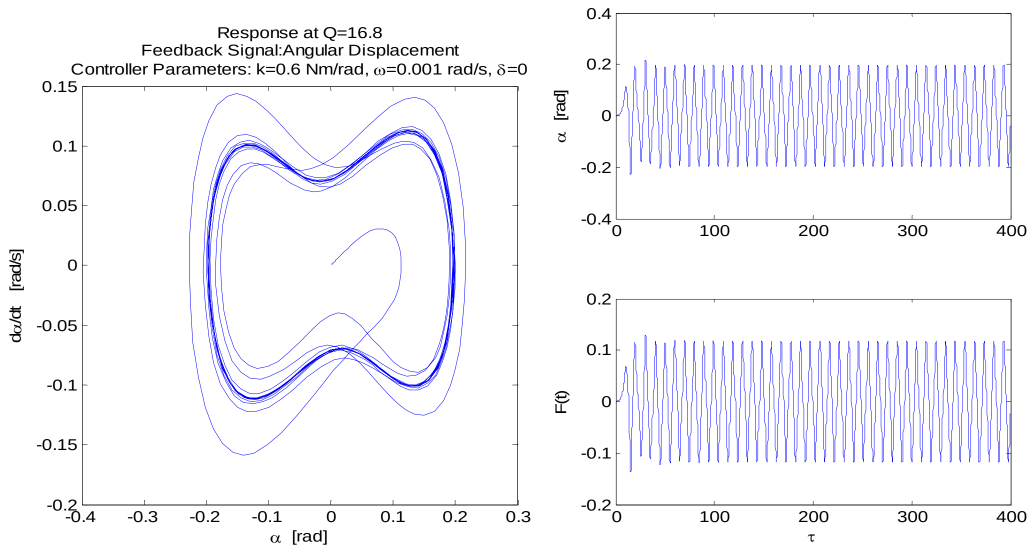

Figure 11. Control on at τ = 0 (Angular Displacement)

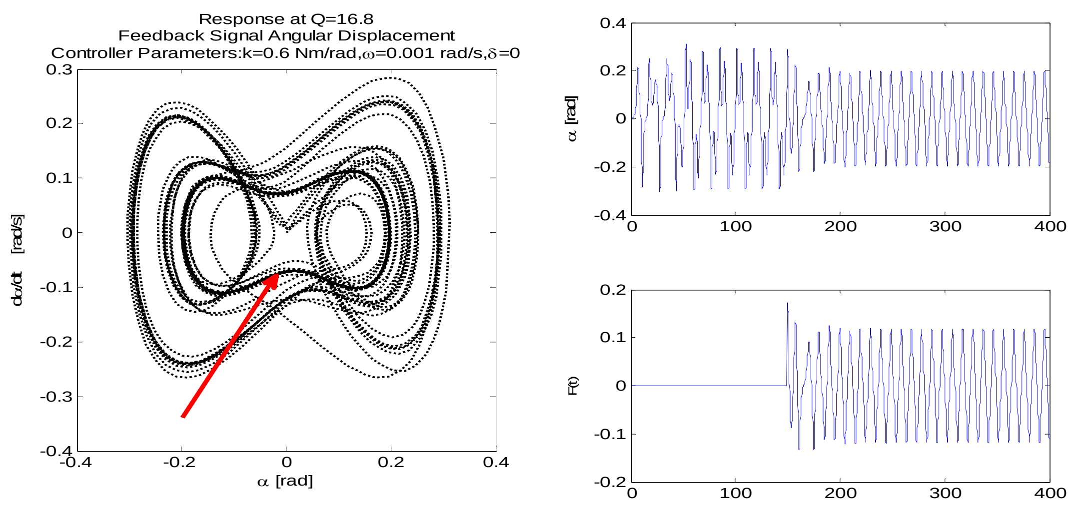

Figure 12. Control on at τ = 150 (Angular Displacement)

Figure 13 is also dynamic stabilization case at k=3 Nm/rad for an ω^c=0.001 rad/s. The controller is again switched on at τ = 150. Note the excursion in the phase space; this likely due the energy being transferred from pitch displacement to the pitch rate. On the other hand, in Figure 14, the feedback signal employed is the angular velocity (pitch rate). This time the controller is engaged at τ = 398. The selected values of τ correspond to one of the zero crossing in the pitch response.

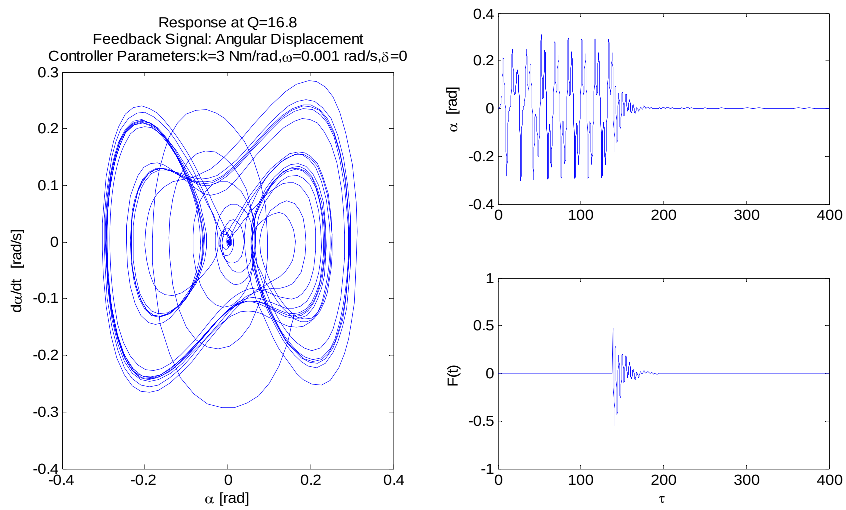

Figure 13. Dynamic control case, τ = 398

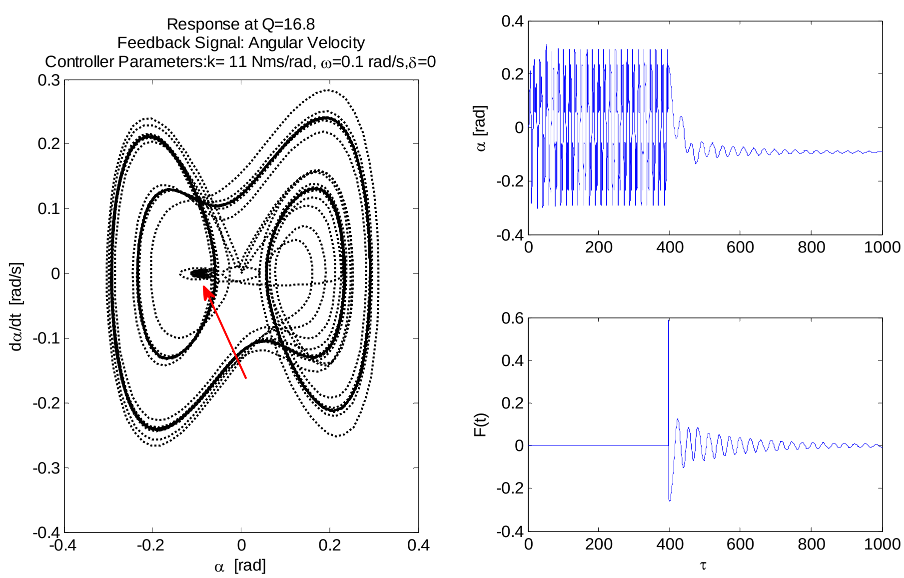

Figure 14. Dynamic control case (Angular Velocity)

The following discusses the contribution of time-delay on the aeroelastic stability of the system. A parameter sweep of the dimensionless time delay constant, δ from zero to 100 with ω^c=0.1 rad/s and k=11 Nm s/rad reveals that there are several instances for which the LCO occurs corresponding to a very specific ranges of δ (Figure 15). The flat regions are regions where a stable response occurs. We see that a single frequency LCO (i.e. no chaotic motion) occurs at time delays δ around the values 24, 50 and 77. These values are sometimes referred to as critical time delay values because it is at these values the stability of the systems changes [8]. Note that for the values of 24, 50 and 77 occur at the same max peak LCO amplitude of 0.22 radians (or 11.46°). How is this information useful in the design of a flutter control system? With this information, a flight dynamicist will be able to alter the elements of the control system such that the inherent time delay falls in one of the flat regions identified by the time delay parameter sweep. The time history of the closed-loop system at δ = 24 and at δ = 35 are reported in Figure 16a and 16b, respectively. As indicated earlier, for the case of δ = 24, the wing structure becomes entrained in a limit cycle.

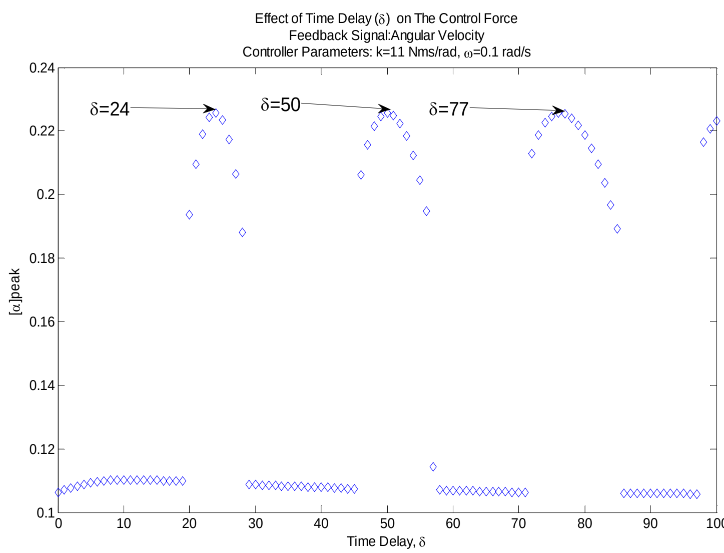

Figure 15. LCO regions associated with time delay domain

CONCLUSIONS

In this study we have examined the application of an adaptive controller augmented with a time delay, for the purposed of stabilizing a nonlinear aeroelastic system in a post flutter flight condition. The results show that the controller is able to transition the aeroelastic system from a chaotic state to limit cycle oscillation. With further increases in the feedback gain and lowering the cutoff frequency of the filter one can suppress fully the limit cycle. We have also shown that of the two feedback signals (pitch and pitch rate) that pitch is most effective as a feedback signal because it requires less control power and does not induce a steady state offset in the angle of attack, unless the low pass filter is not configured properly. In addition, we have shown that we are able to test for regions of instability due to inherent time delay in a nonlinear aeroelastic system.

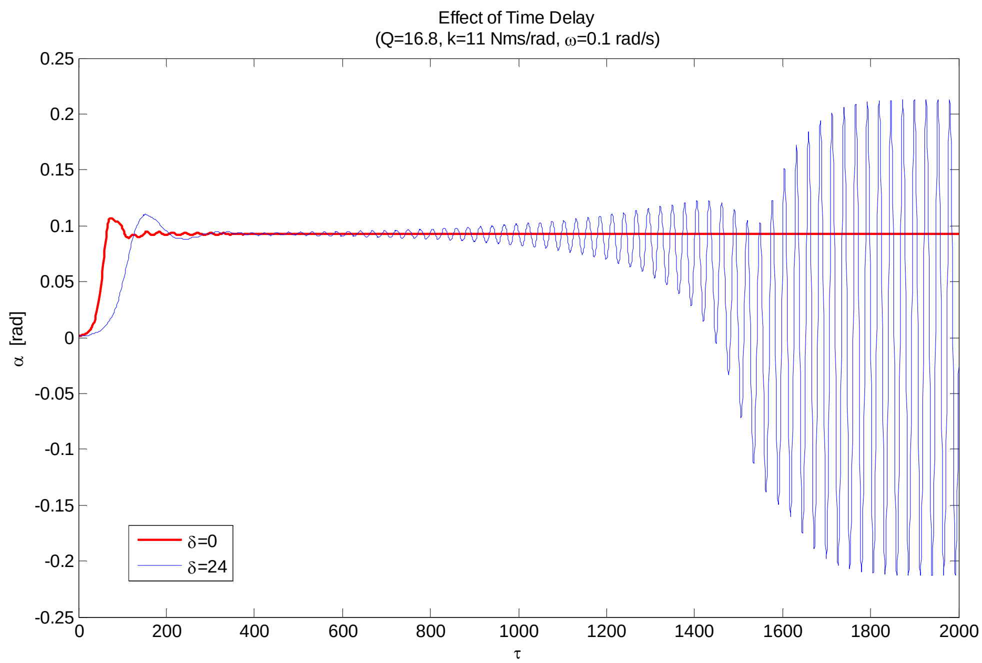

Figure 16a. Time History, δ= 24

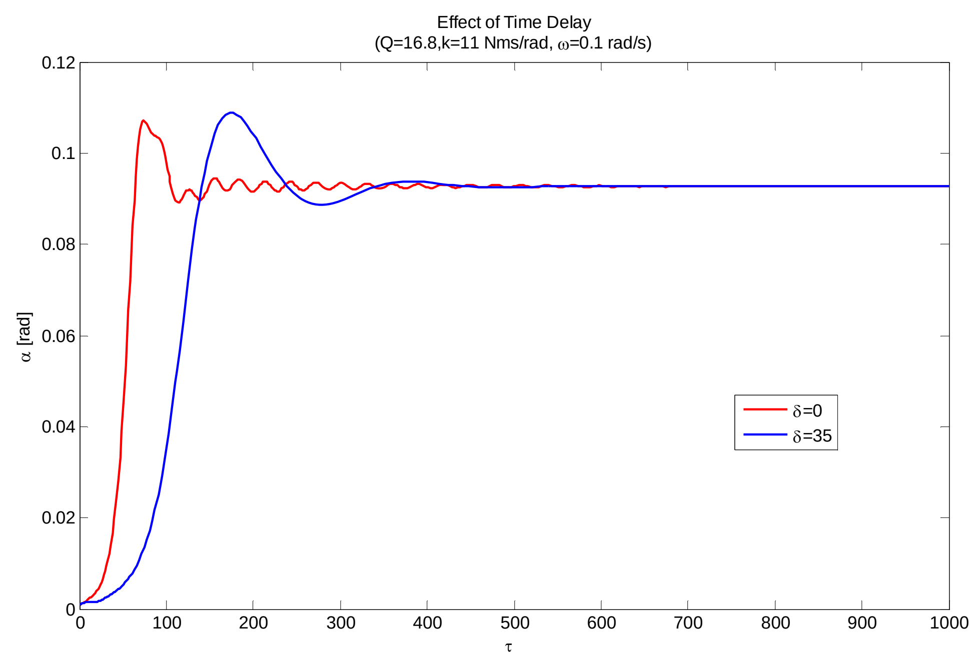

Figure 16b. Time History, δ = 35

Acknowledgments

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