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Mixing, Transport and Coherent Structures

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Abstract. Invariants of topological spaces of dimension three play a major role in many areas, in particular ...

Introduction by the Organisers

The workshop *Invariants of topological spaces of dimension three*, organised by Max Muster (München) and Bill E. Xample (New York) was well attended with over 30 participants with broad geographic representation from all continents. This workshop was a nice blend of researchers with various backgrounds ...

Workshop: Mixing, Transport and Coherent Structures

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Abstracts

Finite Time Curvature and a Differential Geometry Perspective of Shape Coherence by Nonhyperbolic Splitting

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1. Introduction

Recently the notion of coherence has been pushed toward a more rigorous footing, and particularly within the recent advances of finite-time studies of nonautonomous dynamical systems. Here we recall shape coherent sets proved to correspond to slowly evolving curvature, for which tangency of finite time stable and unstable foliations serve a central role. Zero-angle curves, meaning non-hyperbolic splitting, describe boundaries of shape coherent sets. We show a Finite-Time Curvature evolution field (FTC) is particularly useful in identifying curves that correspond to persistent shape coherence.

2. Shape Coherence

We recently introduced a definition concerning coherence called shape coherent sets, motivated by an intuitive idea of sets that "hold together" through finite-time.

Definition 2.1. [1] **Finite Time Shape Coherence** *The shape coherence factor α between two measurable nonempty sets A and B under a flow Φ_t after a finite time epoch $t \in 0 : T$ is,*

$$\alpha(A, B, T) := \sup_{S(B)} \frac{m(S(B) \cap \Phi_T(A))}{m(B)}, \tag{2.1}$$

where $S(B)$ is a group of transformations of rigid body motions of B.

We proved that angle of the finite-time stable and unstable foliations as defined,

$$\theta(z, t) := \arccos \frac{\langle f_s^t(z), f_u^t(z)\rangle}{\|f_s^t(z)\| \|f_u^t(z)\|}, \tag{2.2}$$

corresponds to level curves, and the zero level curves correspond to slowly evolving curvature. Furthermore, such curves can be proved to exist and constructed by the implicit function theorem. Finally considering a Finite-Time Curvature evolution field (FTC), by,

$$l_{\epsilon,v}(x) = \{\hat{x} = x + \epsilon s v, -1 < s < 1\}, \tag{2.3}$$

then curvature growth at a point x over a time epoch is defined,

$$c_T(x) = \lim_{\epsilon \to 0} \sup_{\|v\|=1} \kappa(\phi_T[l_{\epsilon,v}(x)]). \tag{2.4}$$

Level curves of this function corresponding a given low threshold can be shown to correspond to a given significant shape coherence, by use of theorem in [1]. See example of FTC in Fig. 1 and corresponding lowest values corresponding to outlining shape coherent sets of a Rossby wave.

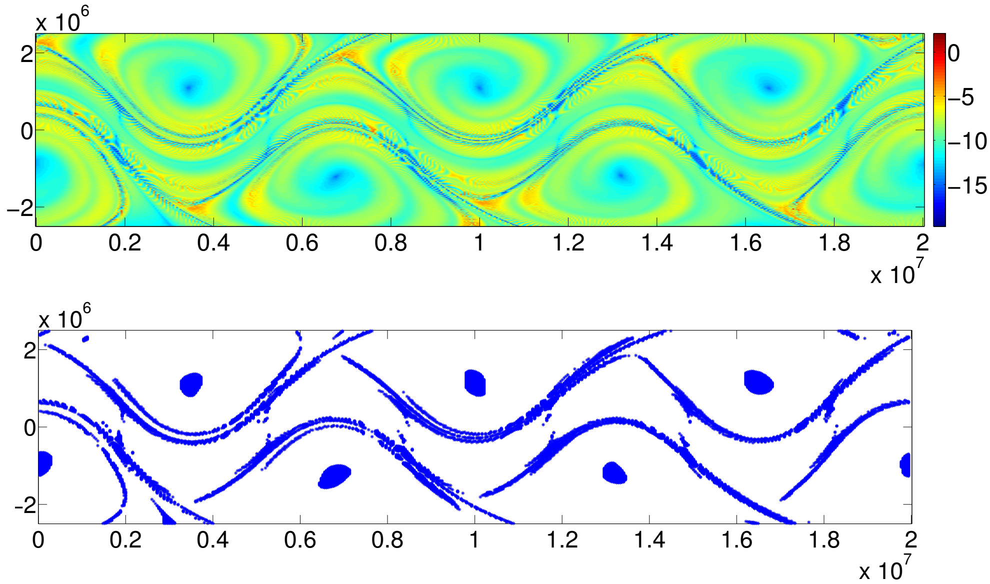

Figure 1. The FTC field of Rossby wave and a low threshold.

References

[1] T. Ma and E. Bollt, *Differential Geometry Perspective of Shape Coherence and Curvature Evolution by Finite-Time Nonhyperbolic Splitting*, (2013) submitted to SIADS.

Reporter: George Haller