BUOYANT VELOCITY OF SPHERICAL AND NON-SPHERICAL BUBBLES/ DROPLETS

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SUMMARY

In jet/plumes that consist of a mixture of oil and gas or oil and hydrate particles, there is a slip velocity between rising bubbles and the surrounding liquid within the jet/plume area. Slip velocity in a bubble plume is the velocity difference between rising bubbles and the surrounding liquid. The value of slip velocity is dependent on the size of individual bubbles. Many previous models of bubble plumes (McDougall, 1978; Fannelop and Sjoen, 1980; Milgram, 1983; Fannelop et al., 1991; Yapa and Zheng, 1997; and Yapa et al., 1999) assumed a constant slip velocity ranging from 0.25 to 0.35 m/s. Some investigators (e.g., Wuest et al., 1992) used empirical formulations for specific gases, such as O_2 in water, but these formulations cannot serve as general formulations for calculating buoyant velocity of particles. In the case of deepwater oil/gas spills, gas bubbles have a wider range of sizes due to the processes of gas hydrate formation/decomposition, dissolution, and expansion. In such models, the effect of bubble size on slip velocity cannot be ignored.

The Two Equation Approach

There are a large number of models that use a two equation approach to calculate the terminal velocity of a particle (solid, liquid, or gas). In this approach the terminal velocity of a particle is estimated by assuming the particle to be spherical and rigid and applying the force balance between buoyancy and drag forces. The equation approach can be summarized by Eqs. 2 and 3 below with the transition given by Eq. 4. For details, please see the complete paper.

\[ U_T = \frac{g d^2 \Delta \rho}{18 \mu} \]  
\[ (2) \]

\[ U_T = \left(\frac{8 g d \Delta \rho}{3 \rho}\right)^{1/2} \]  
\[ (3) \]

\[ d_c = \frac{9.52 \mu^{2/3}}{(g \rho \Delta \rho)^{1/3}} \]  
\[ (4) \]
An Integrated Approach

An integrated formulation is presented to calculate the buoyant velocity of bubbles/droplets of various sizes. The bubble/droplet shape can be a sphere, ellipsoid, or a spherical-cap. The bubbles/droplets can be solids, liquid, or gases. (Usually bubbles/droplets mean gas/liquid, how about “This formulation can be applied to solids, liquid, or gases) The comparison of the calculated results with experimental data shows a good match. The comparison shows that the formulation presented is better than the Stokes law and Reynolds law combination when dealing with bubbles/droplets in a wider range of sizes. The work here was developed in connection with oil and gas spill models that have buoyant oil, gas, or gas-hydrates, although they can be applied to other hydraulic engineering problems as well.

An integrated formulation is presented to calculate the buoyant velocity of bubbles/particles. The formulation is based on various available literature. Many of the equations presented here can be found in the book by Clift et al. (1978), but they are scattered in different chapters and are not organized in a way that is easy to use in hydraulic engineering applications. The merits of the approach presented here are not found in related literature. The integrated approach to estimate the terminal velocity of particles in another fluid can cover a broad range of particle sizes. This approach can be used not only for gas bubbles but also for liquid drops and solid particles. The comparison shows an excellent match between the calculated values and the experimental data. The calculation of terminal velocity of various bubbles/particles is useful for many hydraulic engineering applications such as oil or gas plumes, ultra-deep water gas jets, sediment transport, and ice hydraulic applications.

Figures 1 to 4 compare the results from the two-equation and integrated approaches with the experimental data for various bubble/droplet types and sizes in different ambient conditions. Fig. 1 is for air bubbles in tap water. Fig. 2 is for carbon tetrachloride drops in tap water. Fig. 3 is for carbon dioxide bubbles in 90.6% aqueous glycerol solution. Fig. 4 is for carbon dioxide bubbles in tap water. These four figures show that the results from the integrated approach match very well with the experimental data, while the results from the two equation approach differ from the experimental data as the size grows. The relevant parameters used in the above simulations are listed in Table 3.
Fig. 3 Terminal velocity of CO$_2$ bubbles in 90.6\% aqueous glycerol solution at 25 $^\circ$C

Fig. 4 Terminal velocity of CO$_2$ bubbles in tap water at 22 $^\circ$C