Evaluate the integral: Answer key.

(1) $\int x^2e^{-x^3}dx = -\frac{1}{3}e^{-x^3} + C$

(2) $\int \frac{x}{1+x^2}dx = \frac{1}{2}\ln|1+x^2| + C$

(3) $\int \frac{1}{1+x^2}dx = \tan^{-1}(x) + C$

(4) $\int \frac{1}{1+4x^2}dx = \frac{1}{2}\tan^{-1}(2x) + C$

(5) $\int \frac{1}{1+2x^2}dx = \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\,x) + C$

(6) $\int \frac{x+1}{x^2}dx = \ln|x| - \frac{1}{x} + C$

(7) $\int (x-2)(x-3)dx$
$= \frac{x^3}{3} - 5\frac{x^2}{2} + 6x + C$

(8) $\int \sin x\sqrt{1-\cos x}dx$
$= \frac{2}{3}(1-\cos x)^{3/2} + C$

(9) $\int \tan x dx$
$= -\ln|\cos x| + C$
or $= \ln|\sec x| + C$

(10) $\int xe^x dx = xe^x - e^x + C.$

(11) $\int \ln x dx = x\ln x - x + C$

(12) $\int \cot x dx = \ln|\sin x| + C.$

(13) $\int x\ln x dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

(14) $\int x\cos x dx = x\sin x + \cos x + C.$

(15) $\int \tan^{-1}(x)dx = x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| + C$

(16) $\int \sin^{-1}(x)dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C$

(17) $\int e^x\cos x dx = \frac{1}{2}[e^x\cos x + e^x\sin x] + C$

(18) $\int x^5\sqrt[3]{1+x^3}dx$
$= \frac{x^3}{4}(1+x^3)^{4/3} - \frac{3}{28}(1+x^3)^{7/3} + C$

(19) $\int \frac{e^x}{2+e^x}dx = \ln|2+e^x| + C$

(20) $\int \frac{\cos x - \sin x}{\sin x + \cos x}dx$

$= \ln|\sin x + \cos x| + C$

Hint: $\frac{1}{2}$ angle

(21) $\int \sin^2 x dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$

Hint: $\frac{1}{2}$ angle

(22) $\int \cos^2 x dx$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$

Hint: $1 + \tan^2\theta = \sec^2\theta$

(23) $\int \tan^2 x dx$

$= \tan x + x + C$

(24) $\int \sin x \cos x dx$

$= \frac{\sin^2 x}{2} + C$

or

$= -\frac{\cos^2 x}{2} + C$

or

$= -\frac{1}{4}\cos(2x) + C$

(25) $\int \sin^2 x \cos x dx = \frac{\sin^3 x}{3} + C$

(26) $\int \sin x \cos^2 x dx = -\frac{\cos^3 x}{3} + C$

(27) $\int \sin^3 x \cos^2 x dx = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

(28) $\int \sin^3 x \cos^4 x dx = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

(29) $\int \sin^7 x \cos^3 x dx = \frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C$

(30) $\int \sec x dx = \ln|\sec x + \tan x| + C.$

Answer key : Problem set 1

① $I = \int x^2 e^{-x^3}\,dx$

$u = -x^3$; $du = -3x^2\,dx$

$I = \int e^u \frac{du}{-3}$

$= \frac{-1}{3} e^{-x^3} + C$

② $I = \int \frac{x}{1+x^2}\,dx$

$u = 1+x^2$; $du = 2x\,dx$

$I = \int \frac{1}{u}\frac{du}{2}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|1+x^2| + C$

④ $I = \int \frac{1}{1+4x^2}\,dx$

$u = 2x$; $du = 2\,dx$

$I = \int \frac{1}{1+u^2}\frac{du}{2}$

$= \frac{1}{2}\tan^{-1}(2x) + C$

⑥ $I = \int \frac{x+1}{x^2}\,dx$

$= \int \frac{x}{x^2} + \frac{1}{x^2}\,dx$

$= \int \frac{1}{x} + x^{-2}\,dx$

$= \ln|x| - \frac{1}{x} + C$

⑧ $I = \int \sin x\sqrt{1-\cos x}\,dx$

$u = 1-\cos x$; $du = \sin x$

$I = \int \sqrt{u}\,du$

$= \frac{u^{3/2}}{3/2} + C$

$= \frac{2}{3}(1-\cos x)^{3/2} + C.$

⑨ $I = \int \tan x\,dx$

$= \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$; $du = -\sin x\,dx$

$I = \int \frac{1}{u}\frac{du}{(-1)}$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln|\sec x| + C$

(10) $I = \int x e^x \, dx$

$u = x \qquad dv = e^x dx$

$du = 1 dx \qquad v = e^x$

$I = x e^x - \int e^x \, dx$

$= x e^x - e^x + C$

(11) $I = \int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$I = x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int 1 \, dx$

$= x \ln x - x + C.$

(13) $I = \int x \ln x \, dx$

$u = \ln x \qquad dv = x \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$I = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \frac{x^2}{2} \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + C$

(14) $I = \int x \cos x \, dx$

$u = x \qquad dv = \cos x \, dx$

$du = 1 dx \qquad v = \sin x$

$I = x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + C$

(15) $I = \int \tan^{-1}(x) \, dx$

$u = \tan^{-1}(x) \qquad dv = 1 dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$I = x \tan^{-1}(x) - \int \frac{x}{1+x^2} dx$

$= x \tan^{-1}(x) - \frac{1}{2} \ln |1+x^2| + C$

(16) $I = \int \sin^{-1}(x) \, dx$

$u = \sin^{-1}(x) \qquad dv = 1 dx$

$du = \frac{1}{\sqrt{1-x^2}} dx \qquad v = x$

$I = x \sin^{-1}(x) - \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{I_1}$

Let $I_1 = \int \frac{x}{\sqrt{1-x^2}} dx$

$w = 1 - x^2 \qquad dw = -2x \, dx$

$I_1 = \frac{-1}{2} \int w^{-1/2} dw$

$I_1 = -\frac{1}{2} \cdot 2 \cdot (1-x^2)^{1/2}$

$I = x \sin^{-1}(x) + \sqrt{1-x^2} + C$

Answer key : Problem set 1

(17) $I = \int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x\, dx \qquad v = e^x$

$I = e^x \cos x - \int e^x (-\sin x)\, dx$

$I = e^x \cos x + \underbrace{\int e^x \sin x \, dx}_{I_1}$

$I_1 = \int e^x \sin x \, dx$

I.B.P $\quad u = \sin x \qquad dv = e^x dx$

$du = \cos x \, dx \qquad v = e^x$

$I_1 = e^x \sin x - \int e^x \cos x \, dx$

$I_1 = e^x \sin x - I$

$I = e^x \cos x + e^x \sin x - I$

$2I = e^x \cos x + e^x \sin x$

$I = \frac{1}{2}[e^x \cos x + e^x \sin x] + C$

(18) $I = \int x^5 \sqrt[3]{1+x^3}\, dx$

$I = \int x^3 \cdot x^2 \sqrt[3]{1+x^3}\, dx$

$u = x^3 \qquad dv = x^2 \sqrt[3]{1+x^3}\, dx$

$du = 3x^2 dx \qquad v = \frac{1}{3} \cdot \frac{(1+x^3)^{4/3}}{4/3}$

$I = \frac{x^3}{4}(1+x^3)^{4/3} - \int \frac{1}{4}(1+x^3)^{4/3} \cdot 3x^2 dx$

$I = \frac{x^3}{4}(1+x^3)^{4/3} - \frac{1}{4}\frac{(1+x^3)^{7/3}}{7/3} + C$

(19) $I = \int \frac{e^x}{2+e^x}\, dx$

$u = 2+e^x \; ; \; du = e^x dx$

$I = \int \frac{1}{u}\, du$

$= \ln|2+e^x| + C$

(20) $I = \int \frac{\cos x - \sin x}{\sin x + \cos x}\, dx$

$u = \sin x + \cos x$

$du = \cos x - \sin x \, dx$

$I = \int \frac{1}{u}\, du$

$= \ln|\sin x + \cos x| + C$

(21) $I = \int \sin^2 x \, dx$

$= \int \frac{1}{2}(1 - \cos 2x)\, dx$

$= \frac{1}{2}\left[\int 1\, dx - \int \cos 2x \, dx\right]$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$

(22) $I = \int \cos^2 x \, dx$

$= \int \frac{1}{2}(1 + \cos 2x)\, dx$

$= \frac{1}{2}\left[\int 1\, dx + \int \cos 2x \, dx\right]$

$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$

(23) $I = \int \tan^2 x \, dx$

$= \int \sec^2 x - 1 \, dx$

$= \int \sec^2 x \, dx - \int 1\, dx$

$= \tan x + x + C$

(24) $I = \int \sin x \cos x \, dx$

$u = \sin x \quad du = \cos x \, dx$

$I = \int u \, du$

$= \frac{\sin^2 x}{2} + C$

(27) $I = \int \sin^2 x \cos^2 x \sin x \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$I = \int [1 - u^2] u^2 \frac{du}{(-1)}$

$= -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

(28) $I = \int \sin^3 x \cos^4 x \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$I = \int (1 - u^2) u^4 \frac{du}{(-1)}$

$= -\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + C$

$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$

(29) $I = \int \sin^7 x \cos^3 x \, dx$

$I = \int \sin^7 x \cos^2 x \cos x \, dx$

$u = \sin x \qquad du = \cos x \, dx$

$I = \int u^7 (1 - u^2) \, du$

$= \frac{u^8}{8} - \frac{u^{10}}{10} + C$

$= \frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C$

(30) $I = \int \sec x \, dx$

$I = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$

$I = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) \, dx$

$I = \int \frac{1}{u} \, du$

$= \ln |u| + C$

$= \ln |\sec x + \tan x| + C$