River Dynamics Modeling

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Turbulence Closure

The often used turbulence closure models are based on Boussinesq’s eddy viscosity concept:

\[ \tau_{ij} = \rho \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \]

- **Zero-equation turbulence models**
  - Mixing length model
  - Subgrid model

- **Two-equation turbulence models**
  - Standard k-ε turbulence model
  - RNG k-ε turbulence model
  - Nonequilibrium k-ε turbulence model
  - k-ω turbulence model

- **Other advanced models**: Non-linear k-ε turbulence model, Reynolds stress/flux model, algebraic Reynolds stress/flux model, LES, DNS, etc.
Movable Bed Roughness Formula

Wu and Wang (1999, JHE)
The existing movable bed roughness formulas are applicable only in cases with sediment grains, ripples and dunes.

In general, there are other contributors to the channel roughness, including channel training works, hydraulic structures, vegetation, alternate bars, islands, channel curvature, and alignment.

The most reliable approach to handling the channel roughness is still calibration using the available data measured at the study site.

In the cases where the banks and bed have different roughness features or floodplains exist, composite Manning’s n or conveyance should be used.

Sediment Adaptation Length

- For Bed Load
  - $L_b$: Related to the scales of dominant bed forms and channel geometry

- For Suspended Load
  \[ L_s = \frac{U h}{\alpha \omega_{sk}} \]
  - $\alpha$: Determined by empirical formula such as Armanini and di Silvio’s (1988) method; or given 0.25-1.0.

- For Bed Material Load
  - $L = \max(L_b, L_s)$
Lags between Flow and Sediment

- Lag between local flow and sediment velocities
  - Considered in two-phase flow models, but usually ignored in most models available.

- Depth-averaged velocity difference
  - Considered

- Sediment deposition and erosion at the bed
  - Considered

- Bed form development, etc.
  - Less known and need to be investigated.
Ratio of Depth-Av. Sediment and Flow Velocities

\[ \beta_s = \frac{\int_a^h u_s cdz}{(U_c \int c dz)} \]

- **Concentration**
- **Current velocity**
- **Suspended-load**
- **Bed-load**
Bed-Load Velocity

Modified van Rijn’s (1984) formula

\[
\frac{u_b}{\sqrt{(\gamma_s/\gamma - 1)gd}} = 1.64 T^{0.5}
\]

Francis, 7.5 mm
Luque and van Beek, 0.9 mm
Luque and van Beek, 1.8 mm
Luque and van Beek, 3.3 mm
Lee and Hsu, 1.36 mm
Lee and Hsu, 2.47 mm
Modified van Rijn’s formula

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Wu et al. (2000) Bed Load Formula

\[
\frac{q_{bk}}{p_{bk} \sqrt{(\gamma_s / \gamma - 1) g d_k^3}} = 0.0053 \left[ \left( \frac{n'}{n} \right)^{3/2} \frac{\tau_b}{\tau_{ck}} - 1 \right]^{2.2}
\]
Wu et al. (2000) Suspended Load Formula

\[
\frac{q_{sk}}{p_{bk} \sqrt{\left(\gamma_s / \gamma - 1\right) g d_k^3}} = 0.0000262 \left[\left(\frac{\tau}{\tau_{ck}} - 1\right) \frac{U}{\omega_{sk}}\right]^{1.74}
\]
Single-sized total load
- Ackers-White (1973) formula is good for coarse sediment, not for fine sediment
- Laursen (1958) formula is good for fine sand and silt, not for coarser sediment
- Yang’s (1973, 1984) formula has two sets of coefficients for sand and gravel
- Wu et al. (2000) and Engelund-Hansen (1967) are good for wider size ranges

Sing-sized bed load
- Wu et al. (2000) formula
- Meyer-Peter and Mueller (1948) formula

Single-sized suspended load
- Zhang (1961) formula

Multiple-sized total load
- Wu et al. (2000) formula is the top choice

*: Ultimately, calibration using measurements is the most reliable approach.
Bed Material Sorting

Bed Material Composition in Mixing Layer:

\[
\frac{\partial (\delta_m p_{bk})}{\partial t} = \frac{\partial z_{bk}}{\partial t} + p_{bk}^* \left( \frac{\partial \delta_m}{\partial t} - \frac{\partial z_b}{\partial t} \right)
\]
Bank Erosion

- Planar Failure Method (Osman and Thorne, 1988)

\[
F_d = W_t \sin \beta = \gamma_s \left( \frac{H^2 - y_d^2}{\tan \beta} - \frac{H'^2}{\tan \alpha} \right) \sin \beta
\]

\[
f_s = \frac{F_r}{F_d}
\]

\[
F_r = \frac{(H - y_d) C}{\sin \beta} + \gamma_s \left( \frac{H^2 - y_d^2}{\tan \beta} - \frac{H'^2}{\tan \alpha} \right) \cos \beta \tan \varphi
\]
CCHE1D Simulation Results
Channel Degradation (Newton, 1951)
Degradation using Different $L$
Degradation using Different Mixing Layer Thickness

![Graph showing degradation using different mixing layer thickness.](image-url)
Channel Aggradation

Configuration of Experiment (SAFHL, 1995)
Size Classes
Bed and Water Surface Elevations (m)

- Initial Bed
- Bed Profile Measured at 4hr
- Bed Profile Measured at 16hr
- Bed Profile Measured at 32.4hr
- Water Surface Measured at 32.4hr
- Calculated, $L_s=0.5m$
- Calculated, $L_s=2.0m$
- Calculated, $L_s=7.3m$

Distance downstream of Inlet (m)

Water surface at 32.4hr

Bed at 4hr

Bed at 16hr

Bed at 32.4hr
In-stream Hydraulic Structures

Measuring flume in Goodwin Creek, MS
DEC Low Drop Structure
Erosion in Pa-Chang River

Initial Thalweg in 1995
Thalweg Measured in 1998
Thalweg Cal'ted by SEDTRA Module
Thalweg Cal'ted by Engelund-H. Formula
Thalweg Cal'ted by Wu et al Formula

Distance Upstream of the Estuary (m)

Thalweg Elevation (m)
Erosion Control Analysis

- Initial Thalweg without Structures for Bed Stabilization
- Thalweg after 10 Years without Structures
- Thalweg after 10 Years with 6 Structures
- Thalweg after 10 Years with 18 Structures
- Water Surface after 10 Years with 18 Structures

Distance Upstream of the Estuary (m)

Thalweg Elevation (m)

40 50 60 70 80 90

49000 50000 51000 52000

40 50 60 70 80 90

49000 50000 51000 52000
Danjiangkou Reservoir in Han River, China
Annual Sediment Deposition in Danjiangkou Reservoir

![Bar chart showing annual sediment deposition from 1970 to 1980. The chart compares measured and simulated deposition values. The x-axis represents the years 1970, 1975, and 1980, and the y-axis represents the annual deposition in 10^8 tons.]
Deposition Profile in Danjiangkou Reservoir

Distance upstream from the Dam (km)

Accumulated Deposition (10^8 tons)

- Measured
- Simulated
Three Gorges Reservoir

Main Stream: 756.32 km (368 CSs)
Jialing River reach: 72.17 km (30 CSs)
Wu River reach: 86.8 km (45 CSs)

Sediment sizes: 0.001 ~ 250 mm
Three Gorges Reservoir (Cont’d)

- Jan 1, 2004 ~ Dec 31, 2005

- Station:
  - Cuntan (604.12 km to the dam)
  - Qingxichang (479.3 km to the dam)
  - Wanxian (291.61 km to the dam)
FASTER2D Simulation Results
Hysteresis of Flow and Sediment Transport

Measured vs. calculated bed-load discharges (Run TM07)

Relation of bed-load discharge and flow velocity (Run TM07)

Configuration of Qu’s (2003) experimental setup
High Flow during 1996 Flood in Lower Yellow River (90 km Long)

Huayuankou - Dazhangzhuang

Dazhangzhuang - Jiahetan
Low Flow during 1996 Flood in Lower Yellow River (90 km Long)

Huayuankou - Dazhangzhuang

Dazhangzhuang - Jiahetan
Flow Discharge in Lower Yellow River during 1982 Flood

![Graph showing flow discharge over time with measured and simulated data.](image)
Sediment Concentration in Lower Yellow River during 1982 Flood

![Graph showing sediment concentration over time](https://via.placeholder.com/150)

- Measured values are represented by squares.
- Simulated values are represented by a line.

The graph compares measured and simulated sediment concentration (kg/m³) over time (days).
Water Surface Contours in the Study Reach of East Fork River
Flow Field in a Bend of the East Fork River
Flow Discharge at Outlet in the Study Reach of the East Fork River

Flow Discharges at Outlet (m$^3$/s)

Time (day)
Sediment Discharge at Outlet in the Study Reach of the East Fork River
Helical Flow

Transversal velocity – linear model:

\[ u_n = U_n + b_s I \left( \frac{2z}{h} - 1 \right) \]

At channel centerline \( I = \frac{U_s h}{r} \)

Distribution of \( I \) in a cross section:

\[
\frac{r I}{\beta h U_s} = 1 - \frac{1 - e^{-B/\sqrt{T_a D_I}}}{e^{B/\sqrt{T_a D_I}} - e^{-B/\sqrt{T_a D_I}}} e^{Bn/\sqrt{T_a D_I}} - \frac{e^{B/\sqrt{T_a D_I}} - 1}{e^{B/\sqrt{T_a D_I}} - e^{-B/\sqrt{T_a D_I}}} e^{-B\eta/\sqrt{T_a D_I}}
\]

where \( U_s \) and \( U_n \) are the depth-av. velocities in streamwise and lateral directions, \( \beta \) is a coefficient determining the magnitude of \( I \), \( T_a \) is the adaptation time scale, \( D_I \) is the dispersion coefficient, and \( \eta \) is the dimensionless distance in lateral direction (Wu and Wang, 2004).

An example distribution of \( I \) is shown in the figure.
Transport Angle of Bed Load

*Helical flow effect:*

Engelund (1974)  \[ \tan \delta_b = 7 \frac{h}{r} \]

where \( \delta_b \) is the angle between bed-load and the main flow direction

Odgaard (1986)  \[ \tan \delta_b = \frac{v_b}{u_b} \]

where \( u_b \) and \( v_b \) are the near-bed flow velocities in the x and y directions
Transport Angle of Bed Load

**Bed slope effect:**

Parker (1984)

\[
\frac{q_{bn}}{q_{bs}} = \tan \delta_b + \frac{1 + \alpha_p \mu_c}{\lambda_s \mu_c} \sqrt{\frac{\Theta_c}{\Theta}} \tan \phi
\]

where \( \phi \) is the lateral inclination of the bed, and \( \Theta \) is the Shields number.

Struiksma et al. (1985) and Sekine and Parker (1992)

\[
\frac{q_{bn}}{q_{bs}} = \tan \delta_b - \beta_b \frac{\partial z_b}{\partial n}
\]

where \( z_b \) is the bed level, \( n \) is the lateral direction, and \( \beta_b \) is a coefficient.

Wu (2004)

\[
\begin{align*}
\alpha_{bx,e} &= \frac{\tau'_b \alpha_{bx} + \lambda_0 \tau_c \sin \phi_x / \sin \phi_r}{\tau'_b \alpha_{by} + \lambda_0 \tau_c \sin \phi_y / \sin \phi_r} \\
\alpha_{by,e} &= \frac{\tau'_b \alpha_{by} + \lambda_0 \tau_c \sin \phi_y / \sin \phi_r}{\tau'_b \alpha_{bx} + \lambda_0 \tau_c \sin \phi_x / \sin \phi_r}
\end{align*}
\]

where \( \phi \) is bed slope angle and \( \phi_r \) is the repose angle.
Dispersion of Suspended Load

Longitudinal velocity:

\[ \frac{u_s - U_s}{U_*} = \frac{1}{\kappa} \left( 1 + 2.3 \log \frac{z}{h} \right) \]

or

\[ \frac{u_s}{U_s} = \frac{m + 1}{m} \left( \frac{z}{h} \right)^{1/m} \]

x- velocity:

\[ u = \alpha_1 u_s + \alpha_2 u_n \]

\[ = \alpha_1 \frac{m + 1}{m} U_s \left( \frac{z}{h} \right)^{1/m} + \alpha_2 \left[ U_n + b_s U_s \frac{h}{r} \left( 2 \frac{z}{h} - 1 \right) \right] \]

Concentration distribution: \[ c = Cf(z) \]
Integration of $x$- convection term:

$$
\int_0^h u cdz = \alpha_{11} \frac{m+1}{m} U_s C \int_0^h \left( \frac{z}{h} \right)^{1/m} f(z) dz + \alpha_{12} U_n C \int_0^h f(z) dz
$$

$$
+ \alpha_{12} b_s U_s C \frac{h}{r} \int_0^h \left( 2 \frac{z}{h} - 1 \right) f(z) dz
$$

Using \( \int_0^h f(z) dz = h, \quad \frac{m+1}{m} \int_0^h \left( \frac{z}{h} \right)^{1/m} f(z) dz \approx h \), leads to

$$
D_{sx} = \frac{1}{h} \left( U h C - \int_0^h u cdz \right) = -\alpha_{12} b_s U_s C \frac{1}{r} \int_0^h \left( 2 \frac{z}{h} - 1 \right) f(z) dz
$$

Similarly

$$
D_{sy} = \frac{1}{h} \left( V h C - \int_0^h v cdz \right) = -\alpha_{22} b_s U_s C \frac{1}{r} \int_0^h \left( 2 \frac{z}{h} - 1 \right) f(z) dz
$$
Dispersion of Momentum

\[ D_{xx} = -\rho \left[ \frac{1}{m(m+2)} \alpha_{11} \alpha_{11} U_s^2 + \frac{2b_s}{2m+1} \alpha_{11} \alpha_{12} IU_s + \frac{b_s^2}{3} \alpha_{12} \alpha_{12} I^2 \right] \]

\[ D_{xy} = -\rho \left[ \frac{1}{m(m+2)} \alpha_{11} \alpha_{21} U_s^2 + \frac{b_s}{2m+1} (\alpha_{11} \alpha_{22} + \alpha_{12} \alpha_{21}) IU_s + \frac{b_s^2}{3} \alpha_{12} \alpha_{22} I^2 \right] \]

\[ D_{yy} = -\rho \left[ \frac{1}{m(m+2)} \alpha_{21} \alpha_{21} U_s^2 + \frac{2b_s}{2m+1} \alpha_{21} \alpha_{22} IU_s + \frac{b_s^2}{3} \alpha_{22} \alpha_{22} I^2 \right] \]
Effects of Helical Flow

Calculated velocity contours without and with helical flow effect in Steffler’s 270° bend (Wu and Wang, 2004)

(a). Without helical flow effect

(b). With helical flow effect
Measured vs. calculated velocities at selected cross-sections in Steffler’s 270° bend (Calculations with and without helical flow effect, Wu and Wang, 2004)
FAST3D Simulation Results
Flow depths: (a) Measured by Odgaard and Bergs (1988) and (b) Simulated by Wu et al. (2000)
FAST3D Simulation of Sedimentation Upstream of TGP Dam

Flow pattern upstream of TGP dam simulated using FAST3D model (Fang and Rodi, 2000)
FAST3D Simulation of Sedimentation Upstream of TGP Dam

Flow velocity and bed surface at cross-sections (Fang and Rodi, 2000)
Local Scour near Instream Structures
Complexity of Flows near Structures

- Bow wave
- Wake of pier
- Horseshoe vortex
- Overflow
- Underflow
- Fixed bed
- Movable bed
Significant Local Flow Features

- Localized dynamic pressure
- Horseshoe and other vortices
- Downward flow
- Turbulence intensified locally
- Pressure and shear stress fluctuations
- Flow unsteadiness
- Gravity effect on bed load
- Etc.
Forces on Sediment Particles

Generalized buoyancy force on a particle:

\[ \vec{f}_p = -\frac{1}{6} \pi d^3 \nabla p \]

Effective tractive force per unit bed area in streamwise direction:

\[ \tau_e = \tau'_b - \frac{a \pi}{6} d (\nabla p_d)_s \]

where \( p_d \) is dynamic pressure, \( d \) is sediment diameter, and \( a \) is coefficient assumed as 4/p.
Corrected Critical Shear Stress

\[ \tau_c = K_p K_d K_s \tau_{c0} \]

Dynamic pressure gradient in vertical direction

\[ K_p = 1 + \frac{1}{(\rho_s - \rho)g} \frac{\partial p_d}{\partial z} \]

Downward flow

\[ K_d = 1/(1 + \sin \beta) \]

\( \beta \) is flow impact angle to the bed

Gravity over steep slope

\[ K_s = \sin(\phi - \varphi)/\sin \phi \]

\( \phi \) is repose angle and \( \varphi \) bed angle.
Sediment Transport Capacity

Modified Van Rijn’s formulas

\[
q_{b^*} = 0.053 \left( \frac{\rho_s - \rho}{\rho} g \right)^{0.5} \frac{d^{1.5}}{D_*^{0.3}} \left( \frac{\tau_e}{\tau_c} - 1 \right)^{2.1}
\]

\[
c_{b^*} = 0.015 \frac{d}{bD_*^{0.3}} \left( \frac{\tau_e}{\tau_c} - 1 \right)^{1.5}
\]
Local Scour around a Bridge Pier

Scour depth contour at final time; simulation by FAST3D (Wu, 2007)
Local Scour around a Bridge Pier

Scour depth with time (Wu, 2007)

Scour Depth (m)

Sheppard et al's Run 7

Ettema (D=0.24 m, d_{50}=1.9 mm)

Yanmaz and Altinbilek's Run 3

Lines: Simulated
Symbols: Measured

Scour depth with time (Wu, 2007)
Local Scour around a Bridge Pier

- Cylinder, Sheppard et al.
- Cylinder, Ettema
- Cylinder, Yanmaz and A.
- Square, Yanmaz and A.
- Perfect Agreement

Measured Maximum Scour Depth (m)
Simulated Maximum Scour Depth (m)

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Headcut Migration

Headcut is an abrupt vertical or nearly vertical drop in stream bed, known as knickpoint. It may migrate upstream and cause significant soil erosion and channel instability.

Flow near a headcut

Headcut erosion model (Wu and Wang, 2005)
Headcut Migration

Simulation by Wu and Wang (2005)
Depth-Averaged 2-D Modeling of Local Scour (FASTER2D)
Depth-Averaged 2-D Formulation

Wu and Wang (ICSF-1, 2002):

\[\tau_e = \alpha_t \max\left(\tau_b, -\frac{\pi}{6} f d \rho g \frac{\partial z_s}{\partial s}\right)\]

with

\[f = \begin{cases} 3.4D_*^{-0.3} f_s & D_* < 50 \\ 52.5D_*^{-1} f_s & D_* \geq 50 \end{cases}\]

\[D_* = d \left[ g \left( \frac{\rho_s}{\rho} - 1 \right) / \nu^2 \right]^{1/3}\]

\[\alpha_t = \left( \frac{\sigma}{\sigma_0} \right)^{1/m} \left[ \int_0^\infty x^m e^{-0.5(x-p)^2} \, dx \right]^{1/m} / \left[ \int_0^\infty x^m e^{-0.5(x-p_0)^2} \, dx \right]^{1/m}\]
2-D Model Used (FASTER2D)

- 2-D shallow water equations
- Standard $k-\varepsilon$ turbulence model
- Finite volume method on curvilinear grid
- SIMPLEC algorithm on collocated grid, with Rhie and Chow’s momentum interpolation
- Hybrid, QUICK, HLPA convection schemes
- SIP (Strongly Implicit Procedure)
Note that the erosion pattern looks reasonable but the deposition not.
Validation of 2-D Model

34 cases in total:
6 spur-dikes,
3 square piers,
25 cylindrical piers.
Local Scour Prediction Contest in ICSF-1

Experiment by Briaud et al., Texas A&M (2002)
## Blind 2-D prediction using FASTER2D (Wu and Wang, 2002)

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Measured Max. Scour Depth</th>
<th>Calculated Max. Scour Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1:</strong> 160 mm diameter circular pier placed in clean sand deposit of 0.3 mm in diameter and subjected to a constant velocity of 0.35 m/s and a depth of 0.375 m over a period of one day.</td>
<td>0.183 m</td>
<td>0.182 m</td>
</tr>
<tr>
<td><strong>Case 2:</strong> 160 mm diameter circular pier placed in clean sand deposit of 0.3 mm and subjected to a multi-velocity hydrograph over a period of 4 days (25 m/s in day 1 and 0.35 m/s in day 2, and then each once in days 3 &amp; 4).</td>
<td>0.185 m</td>
<td>0.205 m</td>
</tr>
</tbody>
</table>
Concluding Remarks

• 3-D flow features, such as localized dynamic pressure, downward flow, vorticity and turbulence, need to be considered in simulation of local scour near in-stream structures.

• A modification approach is proposed to extend the existing sediment entrainment functions to rapidly-varied (strongly non-uniform) flow conditions.

• The enhanced 3-D model predicts well the processes of bridge pier scour and headcut migration.

• The 2-D model with a simplified modification predicts reasonably well the maximum erosion depth, but errors exist in the deposition pattern behind the pier.

• The entrainment functions have not been directly validated by lab and field measurement data; all validations reported are indirect in conjunction with numerical models.
Publications Related


